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COSMIC MODELS AND HOLOGRAPHY

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Chapter 1

Resumen

Una creciente evidencia observacional indica que dentro del marco de la cosmología Friedmann-Robertson-Walker estándar, el contenido, la evolución y la dinámica del universo están bien descritos por un modelo de constante cosmológica (Λ) y materia oscura fría (ΛCDM) plano. Este es un modelo de concordancia en el que las partículas bariónicas representan un mero 4% de la materia del universo, mientras que el restante 96%, el llamado sector oscuro, comprende alrededor del 19% de partículas débilmente interactuantes, no relativistas y no bariónicas (materia oscura fría) responsable de la formación de estructura, y 77% de energía oscura. La energía oscura es una substancia que ser comporta como si tuviera presión negativa, cuya presión p_Λ y densidad ρ_Λ satisfacen $\omega_\Lambda = p_\Lambda/\rho_\Lambda < -1/3$, y que completa el balance para hacer al universo espacialmente plano y que es responsable a la vez de la aceleración cósmica actual. Observaciones cosmológicas recientes provenientes de supernovas de tipo Ia, de anisotropías del fondo de radiación de microondas medidas con el satélite WMAP, de estructura a gran escala, de lente gravitatoria débil y del efecto Sach-Wolfe integrado, proporcionan una evidencia abrumadora en favor de esta aceleración de la expansión del universo en el momento presente.

No sorprende pues que tanto la naturaleza como el origen desconocidos de la energía oscura se hayan convertido en un problema fundamental de la física teórica y de la cosmología observacional. La constante cosmológica (o energía del vacío) es el candidato más obvio para tratar este asunto ya que se ajusta bien a los tests cosmológicos que tenemos a nuestra disposición. Sin embargo, los bien conocidos problemas del "ajuste fino" y de la coincidencia asociados con la constante cosmológica son razones suficientes para buscar alternativas.

Si las observaciones futuras revelan que la energía oscura no es la constante cosmológ-

ica, entonces se deberían considerar un número de alternativas, incluyendo escenarios en los que la energía oscura interacciona no gravitacionalmente con otras componentes. Como las interacciones con bariones están fuertemente constreñidas por los experimentos, generalmente se asume que la energía oscura sólo puede interaccionar con la materia oscura. Esta posibilidad se está considerando muy activamente en la literatura y se han propuesto modelos de interacción de energía oscura que explican o al menos alivian el problema de la coincidencia, es decir, el hecho de que las densidades de materia oscura y energía oscura sean del mismo orden hoy. Por otro lado, se ajusta bien a las observaciones actuales.

En esta tesis nos hemos centrado en dos avenidas de ataque que intentan arrojar algo de luz sobre el enigma de la aceleración de la expansión cósmica. El trabajo está dividido en dos líneas principales de investigación. Por un lado, hemos presentado un enfoque al problema de la energía oscura basado en el modelo subcuántico de energía oscura. Por otro lado, nos hemos aprovechado del exitoso modelo de energía oscura holográfica y hemos usado algunos modelos de campos escalares para imitar el comportamiento evolutivo de la energía del vacío dinámica y reconstruir estos modelos holográficos de campos escalares de acuerdo con el conjunto de las observaciones más recientes.

La primera línea de investigación comenzó con el modelo de energía oscura subcuántica original. Este modelo se obtiene al pasar de la descripción lagrangiana de las partículas cuánticas relativistas al lagrangiano de una teoría de campos adecuada en el caso en el que la ecuación de onda de Klein-Gordon se interpreta clásicamente en función de un potencial subcuántico relativista. La teoría de campos resultante se aplicó entonces a la cosmología y se mostrque la versión relativista del potencial subcuántico de Bohm, que se puede asociar con una distribución homogénea e isótropa de partículas, se comporta como si fuera la constante cosmológica responsable de la aceleración actual de la expansión del universo, al menos en el límite donde el potencial del campo se anula.

Hemos extendido este modelo con una nueva densidad lagrangiana que nos ha permitido la construcción de dos nuevos modelos que describen la aceleración actual del universo en el escenario Friedmann-Robertson-Walker plano. Uno de estos modelos se descartó por las observaciones pero el otro presentó una solución que, aunque evolucionaba en la región fantasma $w < -1$, estaba libre de la mayoría de los problemas presentes en los escenarios fantasma, tales como las inestabilidades violentas, las singularidades futuras y las violaciones clásicas de las condiciones de energía. Además, hemos estudiado la termodinámica de estos modelos y su descripción holográfica para finalmente proporcionar una interpretación de la energía oscura en función de la energía de entrelazado del universo.

También hemos mostrado en nuestro modelo que, haciendo desaparecer en el tiempo de coincidencia la masa efectiva asociada con las partículas de materia, una fase previa decelerante dominada por materia puede ser seguida por una fase dominada por energía oscura. Finalmente, hemos obtenido expresiones más generales para estas soluciones cósmicas cuánticas para un número arbitrario de dimensiones espaciales, estudiando la estabilidad de estas soluciones así como la emergencia de ondas gravitacionales.

La segunda línea de investigación se basa en el modelo de energía oscura holográfica, que hemos asumido como la teoría subyacente de la energía oscura, y hemos usado los campos escalares de k-esencia cinética y dilatónico para describirla de forma efectiva. Esto se ha hecho estableciendo una conexión entre la densidad de energía oscura holográfica y las densidades de la k-esencia cinética y del dilatón, respectivamente, en un universo de Friedmann-Robertson-Walker plano. Se han reconstruido sus términos cinéticos así como la dinámica de estos modelos holográficos y se han explorado sus consecuencias cosmológicas.

Para finalizar, hemos considerado una interacción fenomenológica entre la energía oscura holográfica y la materia oscura y hemos usado el campo escalar taquiónico para imitar la evolución de esta energía oscura holográfica interactuante. La correspondencia entre las densidades del campo escalar taquiónico y de la energía oscura holográfica nos ha permitido la reconstrucción del potencial y de la dinámica de este modelo holográfico taquiónico interactuante en un universo de Friedmann-Robertson-Walker plano. Hemos mostrado entonces que este modelo puede describir las expansión acelerada de nuestro universo para un espacio de parámetros dado por los resultados observacionales más recientes.

Chapter 2

Introduction

We shall introduce the dark energy problem and the origins of holography as a motivation for the work done in the following chapters.

2.1 The accelerating universe

The evidence for the accelerated expansion was first provided by SNeIa absolute magnitude versus redshift data [1] and other measurements were later provided by ground-based and space surveys. Current SNeIa data extends to $z \leq 2$ and provides more than 5σ evidence for accelerated expansion.

On the other hand, the positions and amplitudes of acoustic peaks in the Cosmic Microwave Background (CMB) show that the universe is almost spatially flat [2].

These results, combined with the BAO peak measurements, LSS tests [3], and galaxy cluster data, provide an overwhelming evidence that indicates that the content, evolution and dynamics of the universe are well described by a flat (ΛCDM) model. This is a concordance model in which baryonic particles account for a mere 4% of the matter in the universe, whereas the remaining 96%, the so-called dark sector, comprises about 19% of non-relativistic, non-baryonic and weakly interacting particles (Cold Dark Matter) responsible for structure formation, and 77% of dark energy. Dark energy is a fluid with pressure p_Λ and density ρ_Λ that satisfy $\omega_\Lambda = p_\Lambda/\rho_\Lambda < -1/3$ and in the ΛCDM model the dark energy is a cosmological constant Λ with $\omega_\Lambda = -1$ whose energy density does not change in time. Dark energy is a substance that behaves as if it had negative pressure and completes the balance to make the universe spatially flat and drives the current cosmic acceleration.

Also, it redshifts away only very slowly as the universe expands and does not cluster on small scales, becoming dominant only recently, in order not to adversely effect large-scale structure formation and big bang nucleosynthesis. The presence of dark energy also effects CMB anisotropies directly through the integrated Sachs-Wolfe (ISW) effect [5].

Although the ΛCDM model is successful in fitting most data, it has a number of theoretical shortcomings. One the worst issues is the discrepancy of 123 orders of magnitude between the observed value of the cosmological constant and the theoretical value predicted by quantum field theory. This is known as the "fine tuning" problem [6].

Another fact that it is difficult to explain in the ΛCDM model is that today both non-relativistic matter and dark energy have comparable energy densities. This comes as a surprise since the matter and dark energy components scale with redshift differently.

At the beginning of cosmic evolution the universe was radiation dominated, today radiation contributes less than 1% of the total energy density. The contribution of dark energy was negligible in the past, and it has become the dominant component only recently. In the future, it will be the only component.

There is only a short period of time when the energy densities of matter and cosmological constant are comparable and it is why we happen to live in this narrow window of time. This is called the “coincidence” problem [6].

In addition to the two problems mentioned above there are other observational facts that appear to conflict with the ΛCDM model, at possibly more then 2σ confidence level. These are the following:

- High redshift SNeIa data are consistent with spatially-flat ΛCDM. However, it favours models with $\omega_{\mathrm{DE}} < -1$. This would mean that the universe in the past was decelerating faster than the ΛCDM model predicts.
- Large-scale velocity flows have amplitude of 400 km/s, larger than what is expected in a ΛCDM model.
- Cosmological simulations based on the ΛCDM model predict that large voids should be filled with many dwarf dark matter halos. This turns out to be true for very large voids (larger than 10 Mpc). Smaller voids however are observed to be surprisingly empty of dark matter halos. For example, based on ΛCDM we would expect to observe on average 10 dwarf galaxies in our local void, but there are none.

All the problems with the cosmological constant listed above urge as to look for alternatives to the ΛCDM model. That is why this thesis is devoted to the study of some dynamical dark energy models.

2.2 Entropy bounds

According to the second law of thermodynamics, the entropy of a closed system tends to grow towards its largest possible value. Entropy bounds try to address the question of what this maximal value is.

2.2.1 The Bekenstein bound

Bekenstein [7] has suggested that for any weakly gravitating matter system, complete and isolated, in asymptotically flat space, the generalised second law of thermodynamics (GSL) implies the following bound on the entropy S and hence on the information the given system is able to provide:

$$S \leq S_B = 2\pi RE \tag{2.1}$$

where E is the total mass-energy of the matter system and R is the radius of the smallest sphere that fits around the matter system and it is larger than its gravitational radius $R > R_g \equiv 2E$.

Later, González-Díaz showed [8] that the Bekenstein bound implies the existence of a limitation on the entropy-to-surface ratio

$$S \leq \frac{A}{4}. \tag{2.2}$$

The plausibility of the Bekenstein bound was confirmed for numerous weakly gravitating systems [9], but the bound is now known to fail for some gravitationally unstable systems (e.g. [10]). Also, the question of whether the GSL implies the Bekenstein bound remains controversial (see, e.g., [11–15]).

2.2.2 The holographic entropy bound

The most discussed entropy bound is the holographic entropy bound. When taking into account the effect of gravity, based on the black hole entropy relation with horizon area, first 't Hooft and later Susskind [16] used the bound (2.2) to argue that the maximal entropy of a system in bounded by its area A

$$S \leq S_H = \frac{A}{4}. \tag{2.3}$$

That is, the maximal entropy of a system is given by the entropy of the black hole with the same size as the system. The bound is valid for weak or strong self-gravity but fails for rapidly evolving systems.

2.2.3 The covariant entropy bound

A covariant formulation of the Holographic bound was advanced by Bousso in [17]: the entropy S on convergent light-sheets L from a closed surface B will not exceed $\frac{1}{4}$ the area of B,

$$S[L(B)] \leq \frac{A(B)}{4}. \tag{2.4}$$

In flat spacetime convergent light-sheets from a closed surface B cover the entire interior of B and the covariant entropy bound (CEB) is the same as the original holographic bound: the entropy interior to B cannot exceed $\frac{1}{4}$ the area of B. In general spacetimes the convergent light-sheets may not cover the interior of B. Specifically, the light sheets may be terminated by a singularity (such as the big bang) or they may stop converging and start to diverge (in which case they are truncated). In both of these cases the light sheets only cover part of the interior of B and the CEB is weaker than the corresponding holographic bound.

The CEB is applicable to arbitrary spacetimes and valid for strong gravity and rapidly evolving systems.

What Bousso has done is to introduce causal limitations to the regions on which the holographic bound can be applied, and intriguingly this seems to prevent the bound from being violated (at least in the cases studied in [17, 18]).

2.2.4 The causal entropy bound

An improved covariant entropy bound, which is applicable to entropy on space-like hypersurfaces and pasts several critical tests was proposed by Brustein and Veneziano [19]. This bound is called causal entropy bound and is based on the assumption that the maximal black hole in the universe is formed through gravitational collapse of perturbations in the universe, and therefore the "Jeans" scale of the perturbations gives a causal connection scale. For a system with limited self-gravitating behaviour, the Bekenstein bound is the tightest, while in other situations, the causal entropy bound is the strongest.

2.3 Holographic dark energy models

The holographic principle is a conjecture that claims that all the information in a volume can be described by the physics at the boundary of that volume. The maximum entropy in that volume is proportional to its surface area. In the cosmological context, this principle will set an upper bound on the entropy of the universe.

This principle is enlightened by investigations of the quantum properties of black holes. Simply speaking, in a quantum gravity system, the conventional local quantum field theory will break down because it contains too many degrees of freedom which will lead to the formation of black hole, breaking then the effectiveness of the quantum field theory.

Actually, the dark energy problem may be in essence an issue of quantum gravity [20]. However, as a complete theory of quantum gravity has not been established yet, it seems that we have to consider the effects of gravity in some effective field theory in which some fundamental principles of quantum gravity should be taken into account. It is commonly believed that the holographic principle [16] is just a fundamental principle of quantum gravity and has been applied to the dark energy problem, giving rise to the holographic dark energy models.

2.3.1 Holographic dark energy model

With the Bekenstein bound in mind, it seems to make sense to require that for an effective quantum field theory in a box of size L with a short distance cutoff (UV cutoff: Λ), the total entropy should satisfy the relation

$$L^3\Lambda^3 \leq S_{BH} = \pi L^2 M_p^2, \tag{2.5}$$

where M_p is the reduced Planck mass in natural units and S_{BH} is the entropy of a black hole of radius L which acts as a long distance cutoff (IR cutoff: L).

Based on the validity of effective local quantum field theory in a box of size L, Cohen et al. [21] suggested a more stringent bound, requiring that the total energy in a region of size L should not exceed the mass of a black hole of the same size. Therefore, this $UV-IR$ relationship gives an upper bound on the zero point energy density

$$\rho_\Lambda \leq L^{-2} M_p^2. \tag{2.6}$$

This means that the maximum entropy in a box of volume L^3 is

$$S_{max} \approx S_{BH}^{3/4}, \tag{2.7}$$

where S_{BH} is the entropy of a black hole of radius L. The largest L is chosen by saturating the bound in Eq.(2.6) so that we obtain the holographic dark energy (HDE) density

$$\rho_\Lambda = 3c^2 M_p^2 L^{-2}, \tag{2.8}$$

where c is a free dimensionless $O(1)$ parameter and the numeric coefficient is chosen for convenience. The UV cut-off is related to the vacuum energy and the IR cut-off is related to the large scale of the universe. Interestingly, this ρ_Λ is comparable to the observed dark energy density $\sim 10^{-10}eV^4$ if we take L as the Hubble scale H^{-1}, where H is the Hubble parameter at the present epoch $H = H_0 \sim 10^{-33}eV$. The fact that quantum field theory over-counts the independent physical degrees of freedom inside the volume explains the success of this estimation over the naive estimate $\rho_\Lambda = O(M_p^4)$. Therefore, holographic dark energy models have the advantage over other models of dark energy in that they do not need an *adhoc* mechanism to cancel the $O(M_p^4)$ zero point energy of the vacuum.

However, Hsu [22] pointed out that this choice for L gives an equation of state $w = 0$ which does not lead to an accelerated universe. Some authors [23] have insisted on taking $L = H^{-1}$ as the IR cut-off as they consider it more natural. However, in order to explain the acceleration of the expansion of the universe, their model has to necessarily include an interaction between dark energy and dark matter. While there are some indications that suggest that this may be the case, the jury is still out. This model has the advantage that by including a coupling between dark matter and dark energy the coincidence problem may be solved or at least alleviated.

Another possibility would be to use the particle horizon but this does not work either because we get $w > -\frac{1}{3}$ [24]. This led Li [24] to propose that the IR cut-off L should be taken as the size of the future event horizon of the universe

$$R_\mathrm{h}(a) \equiv a \int_t^\infty \frac{dt'}{a(t')} = a \int_a^\infty \frac{da'}{Ha'^2} \,, \tag{2.9}$$

where a is the scale factor of the universe.

This choice not only provides a reasonable value for dark energy but also leads to an accelerated universe. This satisfactory HDE model, which shows a dynamical view of the dark energy, may provide a natural solution to the "fine tuning" problem and may be able to solve the coincidence problem (in combination with inflationary models) as showed in [24]. The HDE model has been tested by various observational data including SNIa [25], SNIa+BAO+CMB [26,27], X-ray gas mass fraction of galaxy clusters [28], differential ages of passively evolving galaxies [29], Sandage-Leob test [30], and so on [31]. These analyses show that the HDE model is consistent with the observational data, being even mildly favoured over the ΛCDM [32].

Given that $\Omega_\Lambda = \rho_\Lambda/\rho_{cr}$ and that $\rho_{cr} = 3M_P^2H^2$ is the critical energy density of our universe, from Eq. (2.8) we see that

$$HR_h = \frac{c}{\sqrt{\Omega_\Lambda}}. \tag{2.10}$$

Differentiating with respect to cosmic time t on both sides of Eq. (2.9), yields

$$\dot{R}_h = HR_h - 1 = \frac{c}{\sqrt{\Omega_\Lambda}} - 1. \tag{2.11}$$

The entropy of our universe is

$$S = \frac{A}{4} = \pi M_P^2 R_h^2. \tag{2.12}$$

In order not to violate the second law of thermodynamics, we require that this entropy does not decrease, we need $\dot{R}_h \geq 0$ which by Eq. (2.11) implies $c \geq 1$. The requirement $c \geq 1$ must be imposed because the universe will be gradually dominated by dark energy, so $\dot{R}_h$ approaches $c - 1$ in the far future. If we take $c = 1$ the behaviour is similar to the cosmological constant.

If $c < 1$, the Gibbons-Hawking entropy would eventually decrease as the future event horizon would shrink, violating the second law of thermodynamics. In fact, it would shrink to zero and the IR cut-off would become shorter than the UV cut-off in a finite time in the future. This would imply that the definition of HDE would break down. On the other hand, in this case we would have dark energy behaving as phantom. In addition to the local arguments against phantom, such as violent quantum instabilities [33,34], Li provided also a global argument against it [24].

In this model there are two model parameters, $\Omega_{m,0}$ and c.

However, some authors claimed that the model has a serious problem concerning causality, because the existence of the future event horizon requires an eternal accelerated expansion of the universe. This led them to propose another two different IR cut-offs. The corresponding holographic models of dark energy obtained as a result are both phenomenologically viable. However,it seems that among these models, the HDE model is more favoured by the observational data [35]. We shall briefly examine these models in the following subsections.

2.3.2 Holographic agegraphic dark energy model

In this model [36] the IR cut-off was first taken to be the age of the universe and then, after some inconsistencies, was finally replaced by the conformal age of the universe [37]

$$\eta \equiv \int \frac{dt}{a} = \int \frac{da}{a^2 H} \ , \tag{2.13}$$

so the energy density in the holographic agegraphic dark energy model is

$$\rho_\Lambda = 3n^2 M_P^2 \eta^{-2}, \tag{2.14}$$

where the new constant parameter n replaces the old parameter c in order to distinguish it from the HDE one. The numerical factor $3n^2$ is introduced to parameterise some uncertainties, such as the species of quantum fields in the universe.

This is a single-parameter model because once n is given, $\Omega_{m,0}$ can be easily obtained. It must be mentioned that only the ΛCDM and the DGP braneworld models share this characteristic.

The coincidence problem could be solved naturally in this model provided that n is of order unity.

2.3.3 Holographic Ricci dark energy model

In this model [38] the IR cut-off was considered to be the average radius of the Ricci scalar curvature, $|R|^{-1/2}$.

For a spatially flat FRW universe, the Ricci scalar is

$$R = -6\left(\dot{H} + 2H^2\right), \tag{2.15}$$

giving the following energy density

$$\rho_\Lambda = 3M_p^2 \alpha \left(H^2 + 2\dot{H}\right) = -\frac{\alpha}{16\pi} R, \tag{2.16}$$

where α is a positive numerical constant to be determined by observations.

This model avoids the causality problem of HDE, because the dark energy is obtained from the locally determined Ricci scalar curvature, not the future event horizon. In addition, it solves the fine tuning problem and the coincidence problem. However, the motivations for such a choice for the IR cut-off were unclear in their first paper [38] although Cai et al. [39] have provided a plausible motivation suggesting that this model can be viewed as originated from taking the causal connection scale as the IR cut-off.

This model is also a two-parameter model, $\Omega_{m,0}$ and α.

A Ricci-like model has also been proposed by Granda and Oliveros [40], whose energy density is

$$\rho_\Lambda = 3M_p^2 \left(\alpha H^2 + \beta \dot{H}\right), \tag{2.17}$$

where α and β and constants which must satisfy the restrictions imposed by the current observational data.

2.4 Entropic force: $UV - IR$ relation and holographic dark energy

The entropic force formalism proposed by Verlinde [41] can be used to derive the $UV - IR$ relationship proposed by Cohen et al. [21], endowing it with an improved theoretical background. In addition, this formalism, when applied to cosmology, implies the existence of an additional term in the Friedmann equation, which can be identified as HDE [42].

Verlinde has suggested that gravity can be explained as an entropic force F where the first law of thermodynamics can be written as

$$F\Delta x = T\Delta S \tag{2.18}$$

which is a equation of force.

Motivated by Bekenstein's entropy bound, Verlinde postulated that when a test particle moves towards a holographic screen, the change of entropy on the holographic screen is proportional to the mass m of the test particle, and the distance Δx between the test particle and the screen:

$$\Delta S = 2\pi m\Delta x. \tag{2.19}$$

If we relate the temperature T that appears in Eq. (2.18) with the acceleration a that an observer experiences in an accelerated frame according to Unruh's prescription

$$T = \frac{a}{2\pi}, \tag{2.20}$$

we can obtain Newton's second law $F = ma$ by using Eq. (2.19) together with (2.18) and (2.20).

On the other hand, if we relate temperature, energy and the number of used degrees of freedom employing the equipartition rule

$$E = \frac{1}{2}NT. \tag{2.21}$$

then Newton's law of gravity $F = Mm/R^2$ can be obtained from Eqs. (2.19), (2.18) and (2.21) together with a formula for N. And using Eqs. (2.19), (2.20) and (2.21), one can obtain a relation between entropy, used bits n, and Newtonian potential Φ:

$$\frac{S}{n} = -\frac{\Phi}{2}. \tag{2.22}$$

2.4.1 $UV - IR$ relation from entropic force

Since the discovery of holography, the UV/IR correspondence has become an important concept in physics concerning gravity. It is conjectured that when gravity is considered, the UV and IR cutoffs of an effective field theory cannot be chosen independently of each other. One can then write the UV cutoff as a function of the IR cutoff as

$$L_{UV} = f(L_{IR}). \tag{2.23}$$

If we now consider how information on the horizon of a Schwarzschild black hole is coarse-grained on a holographic screen, we can apply Eq. (2.22) to finally arrive (see [42]) at

$$f(L_{IR}) = \sqrt{\beta L_p L_{IR}}, \tag{2.24}$$

where L_p is the Planck length and β is a dimensionless numerical constant, which canceled out in Eq. (2.22), thus remains undetermined. The natural value of β is of order unity.

Inserting Eq. (2.24) into Eq. (2.23) yields

$$L_{UV} = \sqrt{\beta L_p L_{IR}} \tag{2.25}$$

which takes the form

$$\Lambda_{UV}^2 = \sqrt{8\pi}\beta M_p \Lambda_{IR} \tag{2.26}$$

in terms of the UV/IR cut-offs of the energy scales. This is Cohen's et al. holographic UV/IR relation [21]. From here, the holographic dark energy $\rho_\Lambda = 3c^2 M_p^2 R_h^{-2}$ can be obtained by noticing that $\rho_\Lambda \sim \Lambda_{UV}^4$ and choosing the IR cutoff as the future event horizon.

2.4.2 Holographic dark energy from entropic force

In addition to ordinary matter, let us assume that our universe has a future event horizon, as indicated by the recent acceleration of the cosmic expansion. Consider a test particle which lies slightly outside a holographic screen, but inside the future event horizon of the universe. We assume the distance between the test particle and the future event horizon (seen from an observer in the centre of the observable universe) be much larger than a Planck length, so that Newtonian approximation is valid. On the other hand, this distance should be also smaller than the size of the observational universe (in order that the holographic screen is also of cosmological size), so that one can investigate the cosmological consequences.

Verlinde pointed out that the energy of the future event horizon which is seen on the holographic screen takes the form

$$E_h \sim N_h T_h \sim R_h, \tag{2.27}$$

where $T_h \sim H \sim 1/R_h$ is the Gibbons-Hawking temperature of the future event horizon and $N_h \sim R_h^2$ is the number of degrees of freedom.

The energy represented on the holographic screen is

$$E = \frac{4\pi R^3}{3}\rho_m, \tag{2.28}$$

where R is the physical radial coordinate of the test particle and ρ_m is the energy density of the matter components.

According to Verlinde, using Eqs. (2.18), (2.19) and (2.21), we find that the energy on the holographic screen induces a force on the test particle towards it, while the event horizon induces another force pointing outward along R. Therefore, the force experienced by the test particle can be expressed as

$$F \sim \frac{mR_h}{R^2} - 4\pi m R\rho_m. \tag{2.29}$$

The term $4\pi R\rho_m$ corresponds to the Newtonian gravity from the matter components. On the other hand, a new term $mR_h/R^2 \equiv F_h$ arises because of the existence of the future event horizon. This new term, which is due to dark energy, creates a repulsive force.

The potential energy for the test particle arising from the effect of the future event horizon is

$$V_h \sim -\frac{R_h m}{R} = -\frac{c^2 m R^2}{2R_h^2}, \tag{2.30}$$

where c is a numerical constant.

As discussed by Verlinde, the entropic force conjecture leads to the Newtonian gravity. The Friedmann equation can be derived from Newtonian gravity as follows:

The total energy of the test particle can be written as $E = m\dot{R}^2/2 + V$, where

$$V \equiv V_m + V_h = -\frac{4\pi G}{3} m\rho_m R^2 - \frac{c^2 m R^2}{2R_h^2}, \tag{2.31}$$

with the physical radius $R = ar$, where a is the scale factor and r is the comoving coordinate of the particle which by definition is a constant. Dividing Eq. (2.31) by $mR^2/2$, we obtain the Friedmann equation

$$3M_p^2 H^2 = \rho_m + \rho_k + 3c^2 M_p^2 R_h^{-2}, \tag{2.32}$$

where $\rho_k \equiv 6M_p^2 E/(mR^2)$ is the effective energy density for the spatial curvature of the universe. Thus holographic dark energy arises as a consequence of the force experienced by a test particle near a holographic screen of a cosmological size with the presence of the future event horizon.

If we now take the time derivative of the holographic dark energy density, this yields

$$\dot{\rho}_\Lambda = -2\rho_\Lambda(H - R_h^{-1}). \tag{2.33}$$

Eqs. (2.32) and (2.33) fully determine the dynamics of holographic dark energy.

2.5 Goals

There is a strong belief that the universe is undergoing an epoch of accelerated expansion. Recent cosmological observations from Type Ia supernovae (SN Ia) [1], Cosmic Microwave Background (CMB) anisotropies measured with the WMAP satellite [2], Large Scale Structure [3], weak lensing [4] and the integrated Sach-Wolfe effect [5] provide an overwhelming evidence in favour of a present accelerating universe. Within the framework of the standard FRW cosmology, this current acceleration requires the existence of a negative pressure fluid, dubbed dark energy, whose pressure p_Λ and density ρ_Λ satisfy $\omega_\Lambda = p_\Lambda/\rho_\Lambda < -1/3$. Unsurprisingly, the unknown nature and origin of dark energy have become a fundamental problem in theoretical physics and observational cosmology. The cosmological constant (or vacuum energy) is the most obvious candidate to address this issue as it complies well with the cosmological tests at our disposal. However, the well known “fine tuning” problem of the cosmological constant and the coincidence problem [6] are enough reasons to look for alternatives.

In this thesis, we shall focus on two avenues of attack that attempt to shed some light on the puzzle of the acceleration of the cosmic expansion. The work is divided into two main lines of research. On the one hand, we present an approach to the dark energy problem based on the sub-quantum dark energy model. On the other hand, we take advantage of the successful holographic dark energy model and use some scalar field models to mimic the evolutionary behavior of the dynamical vacuum energy and reconstruct these holographic models according to the fits of the observational data set.

The first line of research starts with the original sub-quantum dark energy model [43]. This model follows from upgrading the Lagrangian description of quantum relativistic particles to the Lagrangian of a proper field theory in the case that the Klein-Gordon wave equation is classically interpreted in terms of a relativistic sub-quantum potential. The resulting field theory is then applied to cosmology and showed that the relativistic version of the Bohm's subquantum potential which can be associated with a homogeneous and isotropic distribution of particles behaves as if it was a cosmological constant responsible for the current accelerating expansion of the universe, at least in the limit where the field potential vanishes.

We shall extend this model with a new Lagrangian density that will allow the construction of two new models that describe the current acceleration of the universe in a flat FRW scenario. One of these models is ruled out by observations but the other one presents a solution which, although it evolves in the phantom region $w < -1$, it is free from the majority of the problems present in the phantom scenarios, such as violent instabilities, future singularities and classical violations of energy conditions. In addition, we shall study the thermodynamics of these models and the holographic description to finally provide an interpretation of dark energy in terms of the entangled energy of the universe. We shall also show that in our model, by making the effective mass associated with the matter particles to vanish at the coincidence time, a previous decelerating matter-dominated era can be followed by an era dominated by dark energy. Finally, we shall derive more general expressions for these quantum cosmic solutions for any arbitrary number of spatial dimensions, studying the stability of these solutions as well as the emergence of gravitational waves.

The second subject of research is based on the successful holographic dark energy model [24], which we shall assume as the underlying theory of dark energy and shall use the kinetic k-essence and the dilaton scalar fields to effectively describe it. This will be done by establishing a connection between the holographic dark energy density and the kinetic k-essence and dilaton energy densities, respectively, in a flat FRW universe. Their kinetic terms as well as the dynamics of these holographic models will be reconstructed and the cosmological consequences will be explored.

Finally, we shall consider a phenomenological interaction between holographic dark energy and dark matter and shall use the tachyon scalar field to mimic the evolving behaviour of this interacting holographic dark energy model. The correspondence between the tachyon field and the holographic dark energy densities will allow the reconstruction of the potential and the dynamics of this holographic interacting tachyon model in a flat FRW universe. We shall then show that this model can describe the observed accelerated expansion of our universe with a parameter space given by the most recent observational results.

Note: Throughout this thesis we shall use units such that $\hbar = c = k_B = G = 1$ and shall consider a flat Friedmann-Robertson-Walker (FRW) universe unless otherwise stated.

Chapter 3

Bohmian accelerating universe

Starting with the original sub-quantum dark energy model, the current accelerating phase of the evolution of the universe is considered by constructing most economical cosmic models that use just general relativity and some dominating quantum effects associated with the probabilistic description of quantum physics. Two of such models are explicitly analysed. They are based on the existence of a sub-quantum potential and correspond to a generalisation of the spatially flat exponential model of the de Sitter space. In addition, this chapter also discusses the relativistic physics on which the models are based. In this chapter we shall not use natural units, so $\hbar$, G will appear explicitly in the equations. c will appear in some equations and will be taken as $c = 1$ in others but this will not lead to confusion.

3.1 Introduction

As discussed in the previous chapter, for the time being, the concept of dark energy continues posing one of the biggest problems of all physics which, in spite of many attempts and theories intended to solve or at least ameliorate it, has hitherto not found a conclusive outcome. Among such attempts and tentative theories, without trying to be at all exhaustive, we may count what has been dubbed as quintessence, a scalar-field theory satisfying a equation of state $p = w\rho$, with the parameter w being bounded in such a way that $-1 \leq w \leq -1/3$, or its phantom-energy extension for which $w < -1$. Also very popular have been the so-called cosmic generalized Chaplygin gas theories, where the equation of state adopts quite a more exotic structure, or the tachyonic models for dark energy that describe suitable generalisations from the quintessential scalar fields. Besides some rather serious difficulties in trying to fix the observational data, all of the above theories appear to be artificial such as inflaton theories are within the inflationary paradigm. Quite fashionable have also become in the last few years some forms of modified gravity theories in which one does not include any vacuum field but changes instead the gravitational Lagrangian by adding some convenient extra terms that are able to match inflation for large values of the Ricci curvature and describe an accelerating behaviour at the smallest curvatures. Certain of such theories are equivalent to the introduction of quintessence and phantom fields, but all of them suffer from the typical problems associated with having a non Hilbert-Einstein action and may violate some solar-system tests.

From the observational standpoint the rapidly accumulating data coming from supernova Ia luminosity distance measurements, quasar statistics determinations or studies of the fluctuations in the CMB radiation seem all to imply a value for the parameter of the equation of state which becomes each time closer and closer to $w = -1$, which corresponds to a typical cosmological constant, with a certain ever stronger bias towards slightly smaller values. Thus, the realm where our accelerating universe appears to approximately lie on is one that can be expressed as a phantom-like small perturbation of the de Sitter space. Even though one could eventually accommodate the above dark energy and modified gravity models to account for such an observational scenario, that would ultimately appear rather unnatural. Moreover, none of such models can be shown to simultaneously satisfy the following two requirements, (i) exactly predicting what observational data point out in a natural way, and (ii) an economic principle according to which one should not include unnecessary ingredients such as mysterious cosmic fluids or fields nor modifications of the very well tested background theories such as general relativity. The use of scalar fields in quintessence or k-essence scenarios is not with standing quite similar to including an inflaton in inflationary theories for the early universe [44]. Even though, owing to the success of the inflationary paradigm which actually shares its main characteristics with those of the present universal acceleration, many could take this similarity to be a reason enough to justify the presence of a scalar field also pervading the current universe, it could well be that a cosmic Occam's Razor principle would turn out to be over and above the nice coincidence between predictions of usual models for inflation and what has been found in cosmic observations such as the measurement of background anisotropies. After all, the medieval opinion that the simplest explanation must be the correct one has proved to be extremely fruitful so far and, on the other hand, the paradigm of inflation by itself still raises some deep criticisms. Occam's Razor is also against the idea of modifying gravity by adding to the relativistic Lagrangian some convenient extra terms.

Besides general relativity, quantum theory is the other building block which can never be ignored while constructing a predicting model for any physical system. Although it is true that a quantum behaviour must in general be expected to manifest for small-size systems, cosmology is providing us with situations where the opposite really holds. In fact, fashionable phantom models for the current universe are all characterised by an energy density which increases with time, making in this way the curvature larger as the size of the universe becomes greater. In such models quantum effects should be expected to more clearly manifest at the latest times where the universe becomes largest. Thus, it appears that quantum theory should necessarily be another ingredient in our task to build up an economical theory of current cosmology without contravening the Occam's Razor philosophy.

A cosmological model satisfying all the above requirements has been recently advanced [45]. It was in fact constructed using just a gravitational Hilbert-Einstein action without any extra terms and taking into account the probabilistic quantum effects on the trajectories of the particles but not the dynamical properties of any cosmic field such as quintessence or k-essence. The resulting most interesting cosmic model describes an accelerating universe with an expansion rate that goes beyond that of the de Sitter universe into the phantom regime where the tracked parameter of the universal state equation becomes slightly less than -1, and the future is free from any singularity. Such a model will thus describe what can be dubbed a *benigner* phantom universe because, besides being regular along its entire evolution, it does not show the violent instabilities driven by a non-canonical scalar-field kinetic term as by construction the model does not have a negative kinetic term nor it classically violates the dominant energy condition which guarantees the stability of the theory, contrary to what the customary phantom models do. Another cosmic model was also obtained which describes an initially accelerating universe with equation of state parameter always greater than -1, that eventually becomes decelerating for a while, to finally contract down to a vanishing size asymptotically at infinity. The latter model seems to be less adjustable to current observational data.

3.2 The original subquantum dark energy model

In this section we shall review the new interpretation for dark energy based on a Bohmian sub-quantum potential which was first suggested in [45]. Keeping in mind the idea that dark energy should somehow reflect the otherwise unobservable existence of a cosmological substance with an essentially quantum-mechanical nature, and promoting the so-called Bohm's classical interpretation of quantum mechanics [46] to the status of a field theory in a similar way to how it is made from classical relativistic mechanics to finally produce the model of tachyonic dark energy [47], we will thus be able to finally propose simple "classical" models for dark energy which do not necessarily depend on the existence of any potential for the vacuum scalar field, and bring the imprint of their truly quantum origin, formally in much the same way as Bohm's classical interpretation of quantum mechanics does. From the real part of the Klein-Gordon wave equation applied to a quasi-classical wave function $R\exp(iS/\hbar)$, where the probability amplitude R ($P = |R|^2$) and the action S are real functions of the relativistic coordinates, if the classical energy $E = \partial S/\partial t$ and momentum $p = \nabla S$ are defined, one can write [43]

$$E^2 - p^2 + V_{SQ}^2 = m_0^2, \tag{3.1}$$

where m_0 is the rest mass of the involved particle and V_{SQ} is a relativistic sub-quantum potential,

$$V_{SQ}^2 = \frac{\hbar^2}{R}\left(\nabla^2 R - \frac{\partial^2 R}{\partial t^2}\right), \tag{3.2}$$

which should be interpreted according to the Bohm's idea [46] as the hidden sub-quantum potential that accounts for precisely defined unobservable relativistic variables whose effects would physically manifest in terms of the indeterministic behaviour shown by the given particles. From Eq. (3.1) it immediately follows that $p = \sqrt{E^2 + V_{SQ}^2 - m_0^2}$. Thus, since classically $p = \partial L/\partial[\dot{q(t)}]$ (with L being the Lagrangian of the system and q the spatial coordinates which depends only on time t, $q \equiv q(t)$), we have for the Lagrangian

$$L = \int d\dot{q}p = \int dv\sqrt{\frac{m_0^2}{1-v^2} + M^2}, \tag{3.3}$$

in which $v = \dot{q}$ and $M^2 = V_{SQ}^2 - m_0^2$. In the classical limit $\hbar \to 0$, $V_{SQ} \to 0$, and hence we are left with just the classical relativistic Lagrangian for a particle with rest mass m_0. As shown by Bagla, Jassal and Padmanabhan [47], promoting the quantities entering this simple Lagrangian to their field-theory counter-parts allows us to get a cosmological model with tachyonic dark energy. In what follows we shall explore the question of what kind of cosmological models can be derived if we apply a similar upgrading-to-field procedure starting with Lagrangian (3.3). Two limiting situations will be considered. First of all, we shall look at the case of most cosmological interest which corresponds to the limit of small values of the rest mass, $m_0 \to 0$, for which the Lagrangian becomes

$$\begin{aligned} L &\simeq \sqrt{V_{SQ}^2 - m_0^2}\left(\int dv\left(1 + \frac{m_0^2}{2\left(V_{SQ}^2 - m_0^2\right)(1-v^2)}\right)\right) \\ &= \sqrt{V_{SQ}^2 - m_0^2}v + \frac{m_0^2}{\sqrt{V_{SQ}^2 - m_0^2}}\ln\left[\left(\frac{1+v}{1-v}\right)^{1/4}\right]. \end{aligned} \tag{3.4}$$

This Lagrangian is positive definite whenever $V_{SQ} > 0$. For nonzero values of the sub-quantum potential, we can have physical systems with nonzero Lagrangian even for the massless case where $v = 1$ and $m_0 = 0$ simultaneously. This is made possible because the existence of the sub-quantum potential allows us to consider an effective rest mass given by $M \equiv \sqrt{V_{SQ}^2 - m_0^2}$. On the other hand, since the sub-quantum potential V_{SQ} can take on both positive and negative values, the associated field theory can lead to positive or negative pressure, respectively. Choosing $V_{SQ} < 0$ and hence $L < 0$, in the massless case $m_0 = 0$, $v = 1$, we have

$$L = -|V_{SQ}|. \tag{3.5}$$

Generalising to a field theory in the general case $m_0 \neq 0$, $v < 1$ requires the upgrading $q(t) \to \phi$, a field which will thereby depend on both space and time, $\phi(r,t)$, replacing $v^2 \equiv \dot{q}^2$ for $\partial_i\phi\partial^i\phi$ and the rest mass m_0 for a generic potential $V(\phi)$. In the extreme massless case however the Lagrangian (3.5) does not contain any quantity which can be upgraded to depend on ϕ, so that the Lagrangian for the field theory in the massless case is no longer zero, but it is also given by Eq. (3.5).

In what follows we shall regard Lagrangian (3.5) as containing all the cosmological information that corresponds to a universe whose dark energy is given by a positive cosmological constant, provided the field ϕ is homogeneously and isotropically distributed. This can be accomplished if e.g. the sub-quantum potential is interpreted as that potential associated to the hidden dynamics of the particles making of the CMB radiation. Assuming next a perfect fluid form for the equation of state of the cosmic field ϕ, i.e. introducing a stress-energy tensor

$$T_k^i = (\rho + p)u^i u_k - p\delta_k^i, \tag{3.6}$$

where the energy density ρ and the pressure p that correspond to Lagrangian (3.5) are given by

$$\rho = |V_{SQ}|, \quad p = -|V_{SQ}|, \tag{3.7}$$

and the 4-velocity is

$$u_k = \frac{\partial_k\phi}{\sqrt{\partial_i\phi\partial^i\phi}}. \tag{3.8}$$

From Eqs. (3.7) and the conservation equation for cosmic energy, $d\rho = -3(\rho+p)da/a$, it again follows that $\rho = \kappa^2 = |V_{SQ}| = \text{const.}$, so that the resulting Friedmann equation, $\dot{a} = \kappa a/M_P$ (M_P being the Planck mass), yields the expected solution for the scale factor $a = a_0 \exp\left[\kappa(t-t_0)/M_P\right]$. Eqs. (3.7) immediately leads moreover to a characteristic parameter for the perfect fluid state equation which turns out to be constant and given by $w = p/\rho = -1$. We can conclude therefore that if $m_0 = 0$, $v = 1$ (i.e. $V(\phi) = 0$ and $\partial_i\phi\partial^i\phi = 1$ in the field theory) and $V_{SQ} < 0$ one may assume that the observable CMB radiation makes to appear a sub-quantum potential inducing the presence of a pure cosmological constant given by $\Lambda = \kappa = \sqrt{V_{SQ}}$. In case that the rest mass is $m_0 \neq 0$ and very small, there would be a nonzero field-theory potential $V(\phi) \to m_0$ and the sub-quantum medium would correspond to a cosmic dark energy which would behave like some form of a "tracking" quintessential field [48]. In fact, for in such a case we had for negative V_{SQ} and small but nonzero m_0,

$$L = P = -|M|\sqrt{\partial_i\phi\partial^i\phi} - \frac{V(\phi)^2}{4|M|}\ln\left(\frac{1+\sqrt{\partial_i\phi\partial^i\phi}}{1-\sqrt{\partial_i\phi\partial^i\phi}}\right), \tag{3.9}$$

with M being now given by $M \equiv M[V(\phi)] = -\sqrt{V_{SQ}^2 - V(\phi)^2}$. The pressure p is then a definite negative quantity such that $\partial_i\phi\partial^i\phi < 2V(\phi)$ only if $\partial_i\phi\partial^i\phi$ is sufficiently smaller than $\left(\partial_i\phi\partial^i\phi\right)_c$, with

$$\frac{\sqrt{\left(\partial_i\phi\partial^i\phi\right)_c}}{1-\left(\partial_i\phi\partial^i\phi\right)_c} = \ln\left[\frac{1+\sqrt{\left(\partial_i\phi\partial^i\phi\right)_c}}{\sqrt{1-\left(\partial_i\phi\partial^i\phi\right)_c}}\right].$$

The energy density which together with the pressure p enters the equation of state $p = w(\phi)\rho$ would then read

$$\rho = -\frac{V(\phi)^2}{2|M(\phi)|}\left[\frac{\sqrt{\partial_i\phi\partial^i\phi}}{1-\partial_i\phi\partial^i\phi} - \ln\left(\frac{1+\sqrt{\partial_i\phi\partial^i\phi}}{\sqrt{1-\partial_i\phi\partial^i\phi}}\right)\right]. \tag{3.10}$$

We then note that for the considered range of the kinetic term, we always can in fact choose a range for the parameter entering the equation of state which satisfies $0 \geq w(\phi) \geq -1$.

In the limit that the rest mass and the sub-quantum potential take on very similar values, which is the second situation we shall briefly consider, the Lagrangian can be approximated to

$$L \simeq m_0 \int \frac{dv}{\sqrt{1-v^2}} = \frac{1}{2}m_0 \ln\left(\frac{1-\sqrt{1-v^2}}{1+\sqrt{1-v^2}}\right)^{1/2}. \tag{3.11}$$

Such a Lagrangian is negative definite and, if we upgrade the quantities involved in it so that they become field-theory variable, $m_0 \to V(\phi)$, with $V(\phi)$ a classical potential for the scalar field ϕ, and $v^2 \to \partial_i\phi\partial^i\phi$, it would correspond to a negative pressure

$$p = \frac{1}{2}V(\phi)\ln\left(\frac{1-\sqrt{1-\partial_i\phi\partial^i\phi}}{1+\sqrt{1-\partial_i\phi\partial^i\phi}}\right), \tag{3.12}$$

which is definite negative, and a positive energy density

$$\rho = \frac{V(\phi)}{\sqrt{1-\partial_i\phi\partial^i\phi}} - p. \tag{3.13}$$

Thus, for a perfect fluid equation of state $p = w(\phi)\rho$, this would again correspond to a tracking quintessence-like field.

3.3 Relativistic Bohmian backgrounds

In this section we shall consider new fundamental aspects that strengthen the consistency and provide further physical motivation to the general model reviewed in Sec. 3.2. These new aspects concern both the use of a sub-quantum potential model derived from the application of the Klein-Gordon equation, and the background relativistic theory associated with the cosmic quantum models.

3.3.1 The Klein-Gordon subquantum model

We note here that, although for some time in the past it was generally believed that the Klein-Gordon equation was unobtainable from the Bohm formalism [49], in recent years the Klein-Gordon equation has found satisfactory causal formulations. The solution presented in [50] by Horton *et al.* has to introduce the causal description of time-like flows in an Einstein-Riemann space (otherwise the probability current can assume negative values of its zeroth component and is not generally time-like). However, there exists a causal Klein-Gordon theory in Minkowski space [51] where this is achieved by introducing a cosmological constant as an additional assumption which is justified in view of recent observations. Therefore, it makes perfect sense to use a Klein-Gordon equation in our model [45]. Moreover, the nonclassical character of the current whose continuity equation is derived from the purely imaginary part of the expression resulting from the application of the Klein-Gordon equation to the wave function is guaranteed by the fact that one can never obtain the classical limit by making $\hbar \to 0$. Thus, no classical verdict concerning that current of the kind pointed out by Holland [49] can be established. On the other hand, having a material object whose trajectory escapes out the light cone [49] cannot be used as an argument in favour of the physical unacceptability of the model. Quite the contrary, it expresses its actual essentially quantum content, much as the quite fashionable entangled states of sharp quantum theory seemed at first sight violate special relativity and then turned out to be universally accepted. In both cases, physics is preserved because we are not dealing with real signaling. Actually, in Ch. 4, we shall show that our cosmic models can be also interpreted as being originated from the entanglement energy of the whole universe, without invoking any other cause.

3.3.2 Quantum theory of special relativity

Consistent tachyonic theories for dark energy are grounded on special theory of relativity in such a way that all the physics involved at them stems from Einstein relativity. Our cosmic quantum models actually come from a generalisation from tachyonic theories for which the corresponding background relativistic description ought to contain the quantum probabilistic footprint. Thus, in order to check their consistency, viability and properly motivate the models reviewed in Sec. 3.2, one should investigate the characteristics of the quantum relativistic theory on which they are based. In what follows we shall consider in some detail the basic foundations of that background quantum relativity.

Actually, there are two ways of defining the action of a free system endowed with a rest mass m_0 [52]. The first one is by using the integral expression for the Lagrangian $L = \int pdv$, with the momentum p derived from the Hamilton-Jacobi equation, and inserting it in the expression $S = \int_{t_1}^{t_2} Ldt$. The second procedure stems from the definition $S = \beta \int_a^b ds$, where ds is the line element and the proportionality constant $\beta = m_0 c$ is obtained by going to the non-relativistic limit. The strategy followed here is to apply the first procedure to derive an integral expression for S in the case of a Hamilton-Jacobi equation containing an extra quantum term and then obtain the expression for ds by comparing the resulting expression for S with that is given by the second procedure.

As mentioned above, a Hamilton-Jacobi equation with the quantum extra term can be obtained by applying the Klein-Gordon equation to a quasiclassical wave function $\Psi = R(r,t)\exp(iS(r,t)/\hbar)$ [53], where $R(r,t)$ is the quantum probability amplitude and $S(r,t)$ is the classical action. By the second of the above procedures and $L_Q = -m_0 c^2 E(\varphi, k)$, we immediately get for the general spacetime metric

$$ds = E(\phi, k)dt, \tag{3.14}$$

which consistently reduces to the general metric of special relativity in the limit $\hbar \to 0$. If we take the above line element as invariant, then we obtain for time dilation

$$dt = \frac{E(k)dt_0}{E(\varphi, k)}, \tag{3.15}$$

in which $E(k)$ is the complete elliptic integral of the second kind [54].

A key question that arises now is, does the quantum relativistic description and hence our cosmic quantum models satisfy Lorentz invariance? What should be invariant in the present case is the quantity

$$I = ctE\left(\arcsin\sqrt{\frac{c^2t^2 - x^2}{c^2t^2}}, k\right) \tag{3.16}$$

If we chose a given transformation group in terms of hyperbolic or elliptic functions which leaves invariant (such as it happens for Lorentz transformations) the usual relativistic combination $c^2t^2 - x^2 = c^2t'^2 - x'^2$, then we would obtain

$$I = cQ(t', x')E\left(\arcsin \frac{\sqrt{c^2t'^2 - x'^2}}{cQ(t', x')}, k\right), \tag{3.17}$$

where $Q(t', x') \equiv Q(t', x', \Psi)$ is the expression for the transformation of time t in terms of hyperbolic or elliptic functions. It would follow

$$\left(\frac{I}{cQ(t', x')}\right)^{-1} = \frac{\sqrt{c^2t'^2 - x'^2}}{cQ(t', x')}, \tag{3.18}$$

with $()^{-1}$ denoting the inverted function associated to the elliptic integral of the second kind, generally one of the Jacobian elliptic functions or a given combination of them [54]. Thus, the quantity I can only be invariant under the chosen kind of transformations in the classical limit where $k = 1$. Therefore, a quantum relativity built up in this way would clearly violate Lorentz invariance, at least if we take usual classical values for the coordinates.

In order to obtain the wanted transformation equations we first notice that if we take the coordinate transformation formulas in terms of the usual hyperbolic or some elliptic functions of the rotation angle Φ one can always re-express the invariant quantity I of Einstein special relativity in the form

$$I = cQ(t', x')E\left(\arcsin \left(\frac{\sqrt{c^2t'^2 - x'^2}}{cQ(t', x')}\right)^{-1}, k\right). \tag{3.19}$$

From Eq. (3.19) one can write

$$\left(\frac{I}{cQ(t', x')}\right)^{-1} = \left(\frac{\sqrt{c^2t'^2 - x'^2}}{cQ(t', x')}\right)^{-1}$$

and hence

$$I = \sqrt{c^2t'^2 - x'^2} = ct'E\left(\arcsin \left(\frac{\sqrt{c^2t'^2 - x'^2}}{ct'}\right)^{-1}, k\right), \tag{3.20}$$

that is I would in fact have the form of the Einstein relativistic invariant. If we interpret the coordinates entering Eq. (3.20) as quantum-mechanical coordinates, then our quantum expression for the invariant I given by Eq. (3.16) can be directly obtained from the last equality by making the replacement

$$\sqrt{1 - \frac{x^2}{c^2t^2}} = E\left(\arcsin \sqrt{1 - \frac{x_{clas}^2}{c^2t_{clas}^2}}, k\right) \tag{3.21}$$

or

$$\left(\sqrt{1-\frac{x^2}{c^2t^2}}\right)^{-1} = \sqrt{1-\frac{x_{clas}^2}{c^2t_{clas}^2}}, \tag{3.22}$$

where the notation $()^{-1}$ again means inverted function of the elliptic integral of the second kind, and if the coordinates entering the right-hand-side are taken to be classical coordinates, then those on the left-hand-side must still in fact be considered to be quantum-mechanical coordinates. Classical coordinates are those coordinates used in Einstein special relativity and set the occurrence of a classical physical event in that theory. By quantum coordinates we mean those coordinates which are subject to quantum probabilistic uncertainties and would define what one may call a quantum physical event: i.e. that event which is quantum-mechanically spread throughout the whole existing spacetime with a given probability distribution fixed by the boundaries specifying the extent and physical content of the system.

In what follows we will always express all equations in terms of classical coordinates and therefore, for the sake of simplicity, we shall omit the subscript "clas" from them. The equivalence relation given by expressions (3.21) and (3.22) is equally valid for primed and non primed coordinates and should be ultimately related with the feature that for a given, unique time, t or t', the position coordinate, x or x', must be quantum-mechanically uncertain. From the equalities (3.21) and (3.22) for primed coordinates we get then an expression for I' in terms of classical coordinates

$$I' = ct'E\left(\arcsin\frac{\sqrt{c^2t'^2-x'^2}}{ct'}, k\right), \tag{3.23}$$

which shows the required invariance and in fact becomes the known relativistic result $I' = \sqrt{c^2t'^2-x'^2}$ in the classical limit $\hbar \to 0$.

From expressions (3.21) and (3.22) we also have

$$1-\frac{V^2}{c^2} = E(\varphi,k)^2 \to \frac{V}{c} = \sqrt{1-E(\varphi,k)^2} = \tanh\Phi, \tag{3.24}$$

where V is velocity, $\varphi = \arcsin\sqrt{1-\frac{x^2}{c^2t^2}}$ and we have specialised to using the usual hyperbolic functions. Whence $\cosh\Phi = 1/E(\varphi,k)$, $\sinh\Phi = \sqrt{1-E(\varphi,k)^2}/E(\varphi,k)$, and from the customary hyperbolic transformation formulas for coordinates

$$x = x'\cosh\Phi + ct'\sinh\Phi, \quad ct = ct'\cosh\Phi + x'\sinh\Phi, \tag{3.25}$$

we derive the new quantum relativistic transformation equations

$$x = \frac{x' + ct'\sqrt{1-E(\varphi,k)^2}}{E(\varphi,k)}, \quad ct = \frac{ct' + x'\sqrt{1-E(\varphi,k)^2}}{E(\varphi,k)}. \tag{3.26}$$

Had we started with formulas expressed in terms of the Jacobian elliptic functions [54], such that:

$$\frac{V}{c}=\mathrm{sn}(\Phi,k)=\sqrt{1-E(\varphi,k)^2} \tag{3.27}$$

$$x=x'\mathrm{nc}(\Phi,k)+ct'\mathrm{sc}(\Phi,k),\quad ct=ct'\mathrm{nc}(\Phi,k)+x'\mathrm{sc}(\Phi,k), \tag{3.28}$$

then we would have again obtained Eqs. (3.26), so confirming the quantum-mechanical character of the coordinates entering the left-hand-side of Eqs. (3.21) and (3.22). The above derived expressions are not yet the wanted expressions as they still contain an unnecessary element of classicality due to the feature that when using quantum-mechanical coordinates for the derivation of the velocity V setting $x=0$ the unity of the left-hand-side of Eq. (3.21) would correspond to the complete elliptic integral of the second kind $E(k)$ [54]. Thus, we finally get for the transformation equations

$$\begin{aligned} x &= \frac{\left(x'+ct'\sqrt{1-E(\varphi,k)^2}\right)E(k)}{E(\varphi,k)} \\ ct &= \frac{\left(ct'+x'\sqrt{1-E(\varphi,k)^2}\right)E(k)}{E(\varphi,k)}, \end{aligned} \tag{3.29}$$

that are the wanted final expressions in terms of classical coordinates which reduce to the known Lorentz transformations in the classical limit $\hbar\to 0$. From the formula for time transformation we in fact get time dilation to be the same as that (Eq. 3.15) directly obtained from the metric when referring to two events occurring at one and the same point x', i.e.

$$\Delta t=\frac{E(k)\Delta t_0}{E(\varphi,k)}, \tag{3.30}$$

and from that for space transformation the formula for length contraction referred to one and the same time t'

$$\Delta\ell=\frac{E(\varphi,k)\Delta\ell_0}{E(k)}. \tag{3.31}$$

In any case, the quantum effects would be expected to be very small, that is usually k is generally very close to unity for sufficiently large rest masses of the particles.

For the sake of completeness we shall derive in what follows the transformation of velocity components one can also derive from the coordinate transformations (3.29) that, if space and time themselves are subject to the quantum-mechanical uncertainties, they should be now given as

$$\begin{aligned} v_x &= \frac{v_x'+c\sqrt{1-E(\varphi,k)^2}}{1+\frac{v_x'}{c}\sqrt{1-E(\varphi,k)^2}} \\ v_y &= \frac{v_y'E(\varphi,k)}{E(k)\left(1+\frac{v_x'}{c}\sqrt{1-E(\varphi,k)^2}\right)} \end{aligned} \tag{3.32}$$

$$v_z = \frac{v'_z E(\varphi, k)}{E(k)\left(1 + \frac{v'_x}{c}\sqrt{1 - E(\varphi, k)^2}\right)},$$

which reduce once again to the well-known velocity transformation law of Einstein special relativity in the limit $\hbar \to 0$. Even though they are quantitatively distinct of the latter transformation law, Eqs. (3.32) behave qualitatively in a similar fashion and produce the analogous general velocity addition law as in Einstein's special relativity.

We finally turn to the essentials of the relativistic mechanics and find the formulas for momentum and energy that must be satisfied by the cosmic quantum models to be given by

$$p = \frac{\partial L}{\partial v} = \frac{m_0 c\sqrt{1 - k^2\left(1 - \frac{v^2}{c^2}\right)}}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{3.33}$$

$$E = pv - L = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}} \times \left[\frac{v}{c}\sqrt{1 - k^2\left(1 - \frac{v^2}{c^2}\right)} + \sqrt{1 - \frac{v^2}{c^2}}E(\varphi, k)\right]. \tag{3.34}$$

Obviously, these expressions reduce to $p = m_0 v/\sqrt{1 - v^2/c^2}$ and $E = m_0 c^2/\sqrt{1 - v^2/c^2}$, respectively, in the limit $\hbar \to 0$. Moreover, if we set $v = 0$ then $p = V_Q/c$ and $E = m_0 c^2 E(k)$ which become, respectively, 0 and $m_0 c^2$ when $\hbar \to 0$. It follows then that our quantum special relativistic model has the expected good limiting behaviour.

Unless for rather extreme cases the value of parameter k is very close to unity and therefore the corrections to the customary expressions induced by the present model should be expected to be locally very small. However, they could be perhaps detectable in specially designed experiments using extremely light particles.

The main conclusion that can be drawn from the above discussion is that whereas Lorentz invariance appears to be violated in our quantum description if classical coordinates are considered, such an invariance is preserved when one uses quantum coordinates in that description.

3.4 Dark energy without dark energy

In this section we shall look at the current acceleration of the universe by using the same general economical philosophy as in the previous model, even without invoking, moreover, any effects induced at the primordial inflationary period. The sole ingredients which we shall explicitly include, besides general relativity, are the quantum effects on the trajectories of the particles that make up e.g. the background radiation. Such effects will be modeled through the relativistic generalisation of the original sub-quantum potential formalism by Bohm [46] and lead by themselves to an accelerating expansion which, consistently, goes slightly beyond what is predicted by a cosmological constant. Thus, we use a version of the sub-quantum model for dark energy [43] stemming from the analogy with the classically-interpreted Hamilton-Jacobi equation derived from the Klein-Gordon wave equation for a quasi classical wave function $\Psi = R\exp(iS/\hbar)$, i.e.

$$E^2 - p(v)^2 + \tilde{V}_{SQ}^2 = m_0^2, \tag{3.35}$$

where

$$\tilde{V}_{SQ} = \hbar\sqrt{\frac{\nabla^2 R - \ddot{R}}{R}} \tag{3.36}$$

is the sub-quantum potential, $v = \dot{q}(t)$ and $p = \partial\tilde{L}/\partial\dot{q}$, with $\dot{} = d/dt$ and $\tilde{L}$ being the Lagrangian

$$\tilde{L} = \int d\dot{q}p = \int dv\sqrt{\tilde{V}_{SQ}^2 - m_0^2 + \frac{m_0^2}{1-v^2}}. \tag{3.37}$$

As first shown by Bagla, Jassal and Padmanabhan [47] for the fully classical case and later on by González-Díaz [43] for the case in which the Lagrangian contains a sub-quantum potential, upgrading the quantities entering this simple Lagrangian to their field-theory counterparts actually leads to a cosmological tachyonic model which can be used to predict cosmic acceleration. Following [43] we shall replace then the quantity q for a scalar field ϕ, the quantity $\dot{q}^2 \equiv v^2$ for $\partial_i\phi\partial^i\phi \equiv \dot{\phi}^2$ and the rest mass m_0 for the potential $\tilde{V}(\phi)$. With these replacements and leaving $\tilde{V}_{SQ}$ constant for the moment, we can then integrate Eq. (3.37) to have for the field Lagrangian $\tilde{L} = -\tilde{V}(\phi)E\left(x(\phi), k(\phi)\right)$, with $E(x,k)$ the elliptic integral of the second kind, $x(\phi) = \arcsin\sqrt{1-\dot{\phi}^2}$ and $k = \sqrt{1-\tilde{V}_{SQ}^2/\tilde{V}(\phi)^2}$. At first sight one should also up-grade $\tilde{V}_{SQ}$ to depend on ϕ. However, it will be seen later that such a up-grading would lead to a final expression for $\tilde{V}_{SQ}$ which depends only on $\dot{\phi}$, a dependence that disappears because for the present model to avoid divergences it is necessary that $\dot{\phi}$ be constant.

Even though the up-grading-to-field method has been so far used to just suitably motivate the introduction of a cosmic field model, such a method will be in the present scenario shown to be more than a mere motivating procedure devoid of any physical significance [45].

Actually, even after up-grading, the above model can still be interpreted as physically describing pure background radiation equipped with a sub-quantum potential, taking dark energy to be nothing but the effect left in the classical universe by that sub-quantum potential, provided the following two conditions are fulfilled by the field theory that results after up-grading: (1) the field potential $\tilde{V}(\phi)$ is identically equal to zero, and (2) the time derivative of the scalar field becomes $\dot{\phi}^2 = 1$. In fact, since the sub-quantum potential has not been up-graded to any field-depending quantity, if such two conditions are either shown to hold or imposed, then the up-grading process can readily be seen to be equivalent to an identity operation, leaving the original particle theory essentially unchanged; i.e. the radiation particles and the sub-quantum potential can also be regarded as the unique physically relevant ingredients for the model.

We note that in a FRW framework $\dot{\phi}^2 = 1$ necessarily implies $\phi = q$, and hence the second condition amounts to $\tilde{V}(\phi) = \tilde{V}(q) = m_0 = 0$, so that, restoring the speed of light as c, we have $\dot{q}^2 = v^2 = c^2$. In any event, in what follows we shall eliminate any trace of all classical quantities from our model, thereby representing dark energy by solely the sub-quantum potential, a hidden quantity that has not been up-graded and that by itself should necessarily be associated with the particles described by Lagrangian (3.37), not with any field quantity. Thus, the resulting dark-energy scenario would not have any classical analog. It follows that the condition that we have to impose to the scalar field theory derived in the sub-quantum model [43] to satisfy the requirement that dark energy disappears once we erase any trace of the background quantum effects is that the Lagrangian, energy density and pressure turn all out to only depend on the sub-quantum potential and will all vanish in the limit where any possible cosmological constant and the sub-quantum potential are both zero, i.e. $\Lambda \to 0$, $\tilde{V}_{SQ} \to 0$. Using a more appropriate vector field instead of the scalar field ϕ does not make any difference relative to the final results of the model presented here, which turns out to be finally independent of the precise characteristics of the field other than being characterised by the speed of light and a zero rest mass. It will be seen in what follows that such a condition is fulfilled provided that we start with the Lagrangian density

$$L = -V\left(E(x,k) - \sqrt{1-\dot{\phi}^2}\right), \tag{3.38}$$

where again $x = \arcsin\sqrt{1-\dot{\phi}^2}$ and now $k = \sqrt{1 - V_{SQ}^2/V^2}$, with $V \equiv V(\phi)$ the density of potential energy associated to the field ϕ. We do not expect $\tilde{V}_{SQ}$ to remain constant along the universal expansion but to increase like the volume V of the universe does. It is the sub-quantum potential density $V_{SQ} = \tilde{V}_{SQ}/\mathrm{V}$ appearing in Eq. (3.38) what should be expected to remain constant at all cosmic times. In fact, from the imaginary part of the Klein-Gordon equation applied to the wave function Ψ we can get $v \cdot \nabla R - \dot{R}$ and hence the continuity equation for the probability flux $J = \hbar\ \mathrm{Im}(\Psi^*\nabla\Psi)/(m\mathrm{V})$, $\nabla\cdot J - \dot{\mathrm{P}} = 0$, where P is the probability density P = Probability/V. This continuity equation is the mathematical equivalent of a probability conservation law. Up-grading then the velocity v to $\dot{\phi}$ and noting that $\dot{\phi} = \pm 1$ (see later) it follows that $(\nabla^2 R - \ddot{R})/R = (\nabla^2 P - \ddot{P})/(2P)$, with $P = R^2$. Assuming that the particles move locally according to some causal law [46], one can now average Eq. (3.35) with the probability weighting function $P = R^2$, so that one obtains for the averaged sub-quantum potential squared, $\langle \tilde{V}_{SQ}^2\rangle_{av} = \int\int\int dx^3 P\tilde{V}_{SQ}^2 = \hbar^2 \int\int\int dx^3(\nabla^2 P - \ddot{P}) \equiv \hbar^2\left(\langle\nabla^2 P\rangle_{av} - \langle\ddot{P}\rangle_{av}\right)$. Since the universe is isotropic and homogeneous, the corresponding conserved quantity can then be obtained by simply taking $\langle\tilde{V}_{SQ}^2\rangle_{av}^{1/2}/\mathrm{V} = \langle V_{SQ}^2\rangle_{av}^{1/2}$, that is, renaming for the aim of simplicity all the quantities $\langle f^2\rangle_{av}^{1/2}$ involved in the averaged version of Eq. (3.35) as f, we can again derive Eq. (3.38), now with V_{SQ} a constant conserved quantity when referred to the whole volume V of the isotropic and homogeneous universe.

It is easy to see that in the limit of vanishing V_{SQ}, $VE(x,k)$ reduces to $\sqrt{1-\dot{\phi}^2}$ so that the Lagrangian (3.38) vanishes as required. The pressure and energy density are then obtained from Eq. (3.38) to read

$$p_\phi = -V\left(E(x,k) - \sqrt{1-\dot{\phi}^2}\right) \tag{3.39}$$

$$\rho_\phi = V\left(\frac{\sqrt{\dot{\phi}^2 + \frac{V_{SQ}^2}{V^2}(1-\dot{\phi}^2)}\,\dot{\phi}}{\sqrt{1-\dot{\phi}^2}} + E(x,k) - \frac{1}{\sqrt{1-\dot{\phi}^2}}\right), \tag{3.40}$$

where we have considered $V \equiv V(\phi)$. In any case, for a source with parameter $w(t) = p_\phi/\rho_\phi$ we must always have

$$\frac{\dot{\rho}_\phi}{\rho_\phi} = -3H\left(1 + w(t)\right) = \frac{2\dot{H}}{H}. \tag{3.41}$$

By itself this expression can generally determine the solution for the scale factor $a(t)$, provided w is constant. In such a case, we obtain after integrating Eq. (3.41) for the scale factor

$$a = \left(a_0^{3(1+w_0)/2} + \frac{3}{2}(1+w_0)\kappa t\right)^{2/[3(1+w_0)]},$$

in which a_0 is the initial value of the scale factor and κ is a constant. However, we shall not restrict ourselves here to a constant value for the parameter w of the equation of state but leave it as a time-dependent parameter whose precise expression will be determined later on. Combining now Eq. (3.41) with the expression for $w(t)$ we can then obtain an expression for $d(H^{-1})/dt$ by using Eqs. (3.39) and (3.40) as well. Moreover, multiplying Eqs. (3.39) and (3.40) and using Eq. (3.41), a relation between the potential density V and the elliptic integral E can be derived from the Friedmann equation $H^2 = 8\pi G\rho_\phi/3$. These manipulations allow us to finally obtain

$$
\begin{aligned}
E &= -\left[\frac{A(\dot{\phi}, V, V_{SQ})\dot{\phi}\left(1+\frac{3H^2}{2\dot{H}}\right) - 1 - \frac{3H^2\dot{\phi}^2}{2\dot{H}}}{\sqrt{1-\dot{\phi}^2}}\right] \\
&= -\left\{\frac{\frac{3H^2\dot{\phi}^4V_{SQ}^2}{\dot{H}} - \left(\frac{\dot{H}}{4\pi G}\right)^2(1-\dot{\phi}^2) + \dot{\phi}^2V_{SQ}^2(1+\dot{\phi}^2)}{\sqrt{1-\dot{\phi}^2}\left[\left(\frac{\dot{H}}{4\pi G}\right)^2 - \dot{\phi}^2V_{SQ}^2\right]}\right\}
\end{aligned}
\tag{3.42}
$$

with $A(\dot{\phi}, V, V_{SQ}) = \sqrt{\dot{\phi}^2 + \frac{V_{SQ}^2}{V^2}(1-\dot{\phi}^2)}$, and

$$
V = -\frac{2\pi G\sqrt{1-\dot{\phi}^2}}{\dot{H}\dot{\phi}^2}\left[\left(\frac{\dot{H}}{4\pi G}\right)^2 - \dot{\phi}^2V_{SQ}^2\right]. \tag{3.43}
$$

Thus, simple general expressions for the energy density and pressure can be finally derived to be

$$
\rho_\phi = 6\pi G\left(\dot{H}^{-1}H\dot{\phi}V_{SQ}\right)^2 \tag{3.44}
$$

$$
p_\phi = -4\pi G\dot{H}^{-1}\dot{\phi}^2V_{SQ}^2\left(1+\frac{3H^2}{2\dot{H}}\right) = w(t)\rho_\phi, \tag{3.45}
$$

where

$$
w(t) = -\left(1+\frac{2\dot{H}}{3H^2}\right). \tag{3.46}
$$

The Friedmann equation $H^2 = 8\pi G\rho_\phi/3$, derived from the action integral with the Lagrangian (3.38), corresponds to a universe dominated by sub-quantum energy. Using Eq. (3.44) this Friedmann equation leads to

$$
\dot{H} = \pm 4\pi G\dot{\phi}V_{SQ}, \tag{3.47}
$$

with a slowly-varying $w(t)$ that should be quite close, but still less than -1 (that is, the case that current observations each time more clearly are pointing to [55]). We have also

$$
H = \pm 4\pi G\phi V_{SQ} + C_1, \tag{3.48}
$$

with C_1 an integration constant. Note that from Eqs. (3.43) and (3.47) it follows that $V(\phi) = 0$, which is just one of the two conditions required to make consistent our interpretation. Moreover, if we assume that $\dot{\phi}$ is constant (an assumption which would indeed be

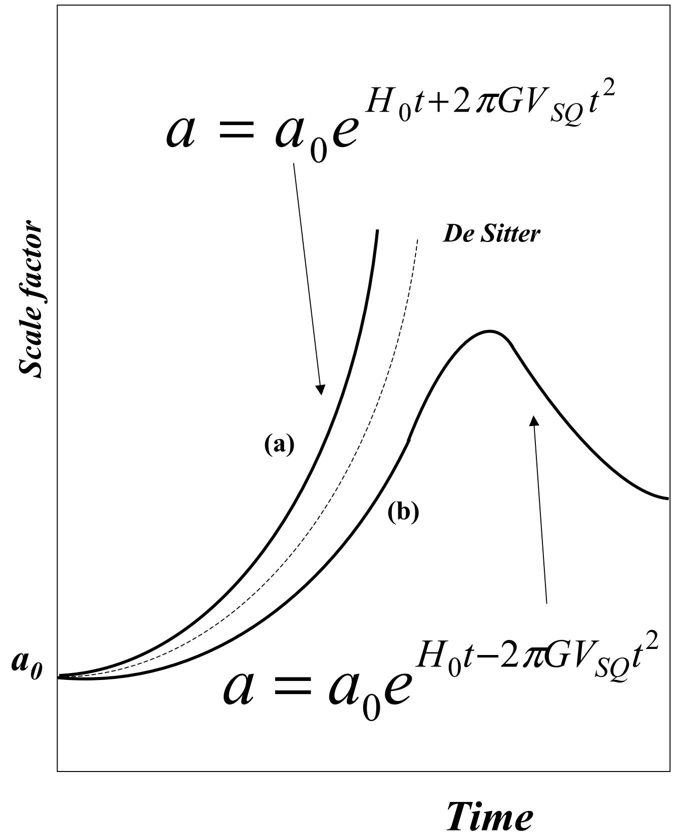

Figure 3.1: Cosmic solutions that result from the introduction of a sub-quantum potential density V_{SQ} when $\dot{\phi}^2 = 1$. Solution (a) goes like in the de Sitter space with the same H_0, but with higher acceleration. Solution (b) corresponds to the case where $H_0^2 > 4\pi V_{SQ}$ and represents a universe which is initially expanding in an accelerated way (at a rate slower than in the de Sitter space with the same H_0), then expands in a decelerated way for a while to finally contract towards a zero radius as $t \to \infty$.

demanded by the fact that $v^2 = 1$ for radiation), then from the equation of motion that corresponds to the Lagrangian for the field ϕ alone [47] $\ddot{\phi} + (1 - \dot{\phi}^2)(3H\dot{\phi} + dV/Vd\phi) = 0$, we have $\dot{\phi}^2 = +1$. Actually, from the Lagrangian density $L_{SQ} = -V(\phi)E(x,k)$ we can also obtain,

$$\dot{\phi}\ddot{\phi} = (1 - \dot{\phi}^2)\{-3H\left[\dot{\phi}^2 + \frac{V_{SQ}^2}{V(\phi)^2}(1 - \dot{\phi}^2)\right] + \sqrt{1 - \dot{\phi}^2}\sqrt{\dot{\phi}^2 + \frac{V_{SQ}^2}{V(\phi)^2}(1 - \dot{\phi}^2)}\frac{\partial L_{SQ}}{V(\phi)\partial\phi} - \frac{\partial V}{V\partial\phi}\dot{\phi}^3\}, \quad (3.49)$$

from which we again derive the conclusion that $\ddot{\phi} = 0$ implies $\dot{\phi}^2 = 1$. Indeed, the assumption that $\dot{\phi}^2 = 1$ can be really regarded as a regularity requirement for $\ddot{\phi}$ at very large ratios $t/\sqrt{4\pi GV_{SQ}}$ or $H_0/\sqrt{4\pi GV_{SQ}}$, because if $\dot{\phi}^2 \neq 1$ then $\ddot{\phi}$ would necessarily tend to diverge at these extreme ratios since $V(\phi)$, by itself, would then tend to vanish even when $\dot{\phi}^2 \neq 1$, as it can be checked from Eqs. (3.43) and (3.47). The same result can then be obtained from the equation of motion derived from Lagrangian (3.38). Hence a vanishing $\ddot{\phi}$ implies that strictly $\dot{\phi}^2 = 1$ and since in addition $V = 0$, once we have down-graded to the original relativistic formalism, we have then that the present model can be interpreted to describe the cosmic sub-quantum effects necessarily associated with an isotropic and homogeneous sea of bosonic particles with zero rest mass which move at the speed of light, i.e. photons - identifying that photon sea with the CMB is just a reasonable assumption. It then follows that the condition $\dot{\phi}^2 = \dot{q}^2 = 1$ becomes a regularity requirement, and the condition $V(\phi) = V(q) = m_0 = 0$ results from the combined effect of the Friedmann equations and the very nature of the model. We have now $\rho_\phi = \rho_q = 6\pi G(\dot{H}^{-1}HV_{SQ})^2 = p_q/w(t)$ which in fact does not depend on any field quantity, such as it was required for interpreting dark energy as the sub-quantum energy associated with radiation particles. The use of an upgrading-to-field motivating method becomes thus rather superfluous in the present theory. We would have indeed obtained identical results and conclusions if we had replaced ϕ and $V(\phi)$ for q and m_0, respectively, leaving V_{SQ} unchanged, in Eqs. (3.38) - (3.48).

It follows then

$$H = \pm 4\pi GV_{SQ}t + C_0, \quad (3.50)$$

in which C_0 is another integration constant, and for the scale factor

$$a_\pm = a_0 e^{\pm 2\pi GV_{SQ}t^2 + C_0 t}. \quad (3.51)$$

Both solutions are depicted in Fig. 1. The solution a_- would predict a universe which initially expands but that immediately starts to contract, tending to vanish as $t \to \infty$. An always accelerating solution slightly beyond the speeding-up predicted by a de Sitter universe is given by the scale factor a_+. In what follows we shall consider the latter solution as that representing the evolution of our current universe and restrict ourselves to deal with that solution only for the branch $t > 0$, denoting $a_+ \equiv a$ and taking then H and $\dot{H}$ to be definite positive.

Thus, the time-dependent parameter of the equation of state will be given by

$$w(t) = -1 - \frac{8\pi G V_{SQ}}{3(4\pi G V_{SQ} t + C_0)^2}, \tag{3.52}$$

which takes on values very close, though slightly less than -1 on the regimes considered in this paper.

Notice that in the limit $V_{SQ} \to 0$, H becomes a constant $H_0 = C_0$, and hence $\rho_\phi \to 3C_0/(8\pi G)$ and $w \to -1$. Clearly, $H_0^2 = \Lambda$ must be interpreted as the cosmological constant associated with the de Sitter solution $a = a_0 e^{H_0 t}$. When we set $C_0 = 0$ instead, then all remaining quantities have the following limiting values

$$\rho_\phi = \frac{p_\phi}{w(t)} = 6\pi G V_{SQ}^2 t^2 \to 0, \tag{3.53}$$

$$w(t) = -1 - \frac{1}{6\pi G V_{SQ} t^2} \to -\infty \tag{3.54}$$

and

$$a = a_0 e^{2\pi G V_{SQ} t^2} \to a_0, \tag{3.55}$$

as $V_{SQ} \to 0$. That is precisely the result we wanted to have and means that all the cosmic speed-up effects currently observed in the universe should be attributed to the purely sub-quantum dynamics that one can associate to the background radiation, rather than to the presence of a dark energy component or any modifications of Hilbert-Einstein gravity. In fact, it can be readily checked that the obtained expression for $\dot{H}$ inexorably leads to a vanishing value for the potential $V(\phi)$, and hence to $\dot{\phi}^2 = 1$, which correspond to pure radiation. Consistency for the present theory is ensured by noticing that: (i) $\dot{\phi}^2 = 1$ does clearly satisfy the Friedmann equation $H^2 = 8\pi G\rho$, with $\rho = 6\pi G(\dot{H}^{-1} H V_{SQ})^2$ and that for the field ϕ from which that condition was derived, and (ii) if we substitute $\dot{\phi}^2 = \dot{q}^2 = 1$ and $V(\phi) = V(q) = 0$ back into Eqs. (3.39) and (3.40) and we use Eqs. (3.42) and (3.43), we recover the regular values for energy density and pressure given by Eqs. (3.44) and (3.45) for $\dot{\phi}^2 = \dot{q}^2 = 1$, which in fact show no dependence whatsoever on any field quantity.

The result that, if there is no cosmological constant term, the considered sub-quantum effects associated with the background radiation will be the responsible for a current accelerating expansion of the universe that goes beyond the cosmological constant limit, implies, on the other hand, that (i) the parameter of the equation of state is necessarily less than -1, though very close to it, (ii) the energy density increases with time, (iii) $\rho_\phi + p_\phi < 0$, that is, the dominant energy condition is violated, and (iv) the kinetic term $\dot{\phi}^2 > 0$. Whereas the first three properties are shared by the so-called phantom models [56], unlike such models, the fourth one guarantees stability of the resulting universe because $V(\phi) = 0$. Also unlike the usual phantom scenarios, the present model does not predict, moreover, any Big Rip singularity in the future. Finally, the considered quantum effects may justify the violation of the dominant energy condition.

3.5 Conclusions

This chapter has dealt with two new four-dimensional cosmological models describing an accelerating universe in the spatially flat case. The ingredients used for constructing these solutions are minimal as they only specify a cosmic relativistic field described by just Hilbert-Einstein gravity and the probabilistic quantum effects associated with particles in the universe. While one of the models is ruled out by the current observational data, the other model corresponds to an equation of state $p = w\rho$ with parameter $w < -1$ for its entire evolution; that is to say, this solution is associated with the so-called phantom sector, showing however a future evolution of the universe which is free from most of the problems confronted by usual phantom scenarios; namely, violent instabilities, future singularities and classical violations of energy conditions. Therefore we have named our phantom model a *benigner phantom* model.

Observational data are being accumulated that each time more accurately point to an equation of state for the current universe which corresponds to a parameter whose value is very close to that of the case of a cosmological constant, but still being less than -1. It appears that one of the models considered in this chapter would adjust perfectly to such a requirement, while it does not show any of the shortcomings that the customary phantom or modified-gravity scenarios now at hand actually have.

Chapter 4

Benigner phantom cosmology: thermodynamics and holography

This chapter discusses the thermodynamics of the two cosmic solutions obtained in Ch. 3, using the second principle as a guide to choose which among the two is more feasible. In addition, the holographic description and an interpretation of dark energy in terms of the entangled energy of the universe are also presented. In this chapter we shall not use natural units, so $\hbar$, G and k_B will appear explicitly in the equations.

4.1 Introduction

We know very little about the theoretical nature and origin of dark energy. Therefore, it is worth exploring its thermodynamic properties seeking a deeper understanding, in the hope that this consideration will shed some light on the properties of dark energy and help us understand its rather elusive nature. Actually, some attention has been paid to the subject of thermodynamics of dark energy when this is interpreted as a radiation field [57] and a phantom field [58]. Other authors have also studied a variety of dark energy properties related to thermodynamics [59–63]. In this chapter we are going to deal with a fundamental aspect of the benigner phantom scenario introduced in the previous chapter. Namely, we shall consider some thermodynamical aspects of it, putting special emphasis on general functions such as entropy, enthalpy as well as temperature. In addition, we shall study the implied holographic description and finally the interpretation of the models in terms of the entanglement energy of the accelerating universe.

4.2 Thermodynamics

The thermodynamical description of dark energy has offered an alternative route to investigate the evolution of the current universe [57]- [63]. However, whereas well-defined expressions can be obtained for dark energy models with equations of state $p = w\rho$ where $w > -1$, in the phantom regime characterised by $w < -1$ there are violent instabilities and a future singularity, the Lagrangian contains a nonphysical kinetic term, and either the temperature or the entropy must be definite negative. In what follows we shall discuss the thermodynamical properties of what we can call the benigner phantom cosmic models in which it will be seen that these problems are largely alleviated. By using the equations for the pressure and the energy density given in the previous chapter we proceed now to derive expressions for the thermodynamical functions according to the distinct models implied by the sign ambiguity in Eq. (3.51) and the possibility that the cosmological term be zero or not, only for the solution branches that correspond to a positive time $t > 0$. On the one hand, the translational energy that can be associated with the scalar field would be proportional to [58] $a^3\dot{\phi}^2$ and therefore, because $\dot{\phi}^2 = 1$ [45], the essentially quantum temperature associated with the sub-quantum models must be generally given by

$$T_{SQ} = \kappa a^3, \tag{4.1}$$

with κ a given positive constant whose value will be determined later. It is worth noting that, unlike for phantom energy models [56], in this case the temperature is definite positive even though the value of the state equation parameter w be less than -1. Moreover, this temperature is an increasing function of the scale factor and hence it will generally increase with time. It must be also stressed that T_{SQ} must be a quantum temperature as it comes solely from the existence of a sub-quantum potential.

On the other hand, one can define the entropy and the enthalpy. If, since the universe evolves along an irreversible way, following the general thermodynamic description for dark energy [57, 58], one defines the total entropy of the sub-quantum medium as $S_{SQ}(a) = \rho\mathrm{V}/T_{SQ}$, with $\mathrm{V} = a^3$ the volume of the universe, then in the case that we choose for the scale factor the simplest expanding solution (without cosmological constant) $a_+ = a_0 \exp(2\pi G V_{SQ} t^2)$, with V_{SQ} the sub-quantum potential density, we obtain the increasing, positive quantity

$$S_{SQ}(a_+) = \frac{V_{SQ}}{\kappa} \ln\left[\left(\frac{a_+}{a_0}\right)^3\right]. \tag{4.2}$$

This definition of entropy satisfies the second law of thermodynamics.

For the kind of systems we are dealing with one may always define a quantity which can be interpreted as the total enthalpy of the universe by using the same expression as for entropy, but referred to the internal energy which, in the present case, is given by $\rho + p$, instead of just ρ. Thus, we can write for the enthalpy $H_{SQ} = (\rho + p)\mathrm{V}/T_{SQ}$. which leads for the same cosmic solution to the constant, negative definite quantity

$$H_{SQ}(a_+) = -\frac{V_{SQ}}{\kappa}, \tag{4.3}$$

whose negative sign actually implies a quantum violation of the dominant energy condition and indicates that we are in the phantom regime.

The consistency of the above definitions of entropy and enthalpy will be guaranteed in what follows because the expressions that we obtain from them in the limit $V_{SQ} \to 0$ are the same as for the de Sitter space.

Since the third power of the ratio a_+/a_0 must be proportional to the number of states in the whole universe, the mathematical expression of the entropy given by Eq. (4.2) could still be interpreted to be just the statistical classical Boltzmann formula, provided we take the constant V_{SQ}/κ to play the role of the Boltzmann constant k_B, or in other words, k_B is taken to be given by $k_B = V_{SQ}/\kappa$, in such a way that the temperature becomes $T_{SQ}(a_+) = V_{SQ}a^3/k_B$ which consistently vanishes at the classical limit $\hbar \to 0$. If we let $\hbar \to 0$ then it would be $T_{SQ}(a_+)$ but not $S_{SQ}(a_+)$ what vanishes. In this way, Eq. (4.3) becomes

$$H_{SQ}(a_+) = -k_B. \tag{4.4}$$

The negative value of this enthalpy can be at first sight taken as a proof of an unphysical character. However, one could also interpret $H_{SQ}(a_+)$ the way Schrödinger did [64] with the so-called "negentropy" as a measure of the information available in the given system, which in the present case is the universe itself.

The above results correspond to the case in which the universe is endowed with a vanishing cosmological constant. If we allow now a nonzero cosmological term H_0 to exist, i.e. if we first choose the solution $a_- = a_0 \exp(H_0 t - 2\pi G V_{SQ} t^2)$, then we have for the expressions of the entropy and enthalpy that correspond to a universe which, if $H_0 > \sqrt{4\pi G V_{SQ}}$, first expands in an accelerated way with $w > -1$, then expands in a decelerating way to finally progressively contract all the way down until it fades out at an infinite time,

$$S_{SQ}(a_-, H_0) = \frac{3H_0^2}{8\pi G\kappa} - \frac{V_{SQ}}{\kappa} \ln\left[\left(\frac{a_-}{a_0}\right)^3\right], \tag{4.5}$$

and again for this case

$$H_{SQ}(a_-, H_0) = \frac{V_{SQ}}{\kappa} = k_B, \tag{4.6}$$

which is now positive definite.

Eq. (4.5) contains two different terms. The first term, $S_{dS} = 3H_0^2 k_B/(8\pi G V_{SQ})$, corresponds to a de Sitter quantum entropy which diverges in the classical limit $\hbar \to 0$. The second one is the same as the statistical-mechanic entropy in Eq. (4.2) but with the sign reversed. It would be worth comparing the first entropy term with the Hawking formula for the de Sitter space-time which is given by the horizon area in Planck units, $S_H \propto H_0^{-2} k_B/(\ell_P^2)$ [65]. At first sight the entropy term S_{dS} appears to be proportional to just the inverse of the Hawking formula. However, one can re-write S_{dS} as $S_{dS} = k_B/(2GH_0\bar{V}_{SQ})$, where $\bar{V}_{SQ} = V_{SQ}\mathrm{V}_{dS}$, with V_{dS} the equivalent volume occupied by de Sitter space-time with horizon at $r = H_0^{-1}$. Now, $\bar{V}_{SQ}$ is the amount of sub-quantum energy contained in that equivalent de Sitter volume, so that we must have $\bar{V}_{SQ} = \hbar H_0$. It follows that S_{dS} actually becomes given by the horizon area in Planck units, too. It is worth noticing that the temperature $T_{SQ}(a_-, H_0)$ can similarly be decomposed into two parts one of which is given by the Gibbons-Hawking expression [65] $\hbar H_0/k_B$, and the other corresponds to the negative volume deficit that the factor $\exp(-2\pi G V_{SQ} t^2)$ introduces in the de Sitter space-time volume.

We note that also for this kind of solution a universe with $T_{SQ}(a_-, H_0) = V_{SQ} a_0^3/k_B$ and $S_{SQ}(a_-, H_0) = S_{dS}$ is left when we set $t = 0$. If we let $\hbar \to 0$, then $T_{SQ}(a_-, H_0) \to 0$ and $S_{SQ}(a_-, H_0) \to \infty$. On the other hand, it follows from Eq. (4.5) that, as the universe evolves from the initial size a_0, the initially positive entropy $S_{SQ}(a_-, H_0)$ progressively decreases until it vanishes at a time $t = t_* = H_0/(4\pi G V_{SQ})$, after which the entropy becomes negative. This would mean a violation of the second law of thermodynamics even on the current evolution of the universe which is induced by quantum effects. Therefore the model that corresponds to Eqs. (4.5) and (4.6) appears to be prevented by the second law.

Finally, we consider the remaining solution $a_+ = a_0 \exp(H_0 t + 2\pi G V_{SQ} t^2)$ which predicts a universe expanding in a super-accelerated fashion all the time up to infinity with $w < -1$. In this case we obtain

$$S_{SQ}(a_+, H_0) = \frac{3H_0^2}{8\pi G\kappa} + \frac{V_{SQ}}{\kappa} \ln\left[\left(\frac{a_+}{a_0}\right)^3\right], \tag{4.7}$$

with $3H_0^2/(8\pi G\kappa) = 3H_0^2 k_B/(8\pi G V_{SQ}) \propto S_H$, and

$$H_{SQ}(a_+, H_0) = -\frac{V_{SQ}}{\kappa} = -k_B. \tag{4.8}$$

All the above discussion on the relation of the sub-quantum thermodynamical functions with the Hawking temperature and entropy holds also in this case, with the sole difference that now $S_{SQ}(a_+, H_0)$ and $T_{SQ}(a_+, H_0)$ are larger than their corresponding Hawking counterparts. Again for this solution a universe with $T_{SQ}(a_+, H_0) = \kappa a_0^3$ and $S_{SQ}(a_+, H_0) = S_{dS}$ is left when we set $t = 0$ whereas $T_{SQ}(a_+, H_0) \to 0$ and $S_{SQ}(a_+, H_0) \to \infty$ in the classical limit $\hbar \to 0$. Moreover, such as it happens when $H_0 = 0$, there is here no violation of the second law for $S_{SQ}(a_+, H_0)$, but $H_{SQ}(a_+, H_0)$ is again a negative constant interpretable like a negative entropy that would mark the onset of existing structures in the universe which are capable to store and process information [64].

In any case, we have shown that the thermodynamical laws derived in this chapter appear to preclude any model with $w > -1$ and so leave only a kind of phantom universe with $w < -1$ as the only possible cosmological alternative compatible with such laws. That kind of model does not show however the sort of shortcomings, including instabilities, negative kinetic field terms or the future singularity named Big Rip, that the usual phantom models have [33,34]. Since we have dealt with an essentially quantum system, the violation of the dominant energy condition that leads to the negative values of the enthalpy H_{SQ} in the thermodynamically-allowed models appears to be a rather benign problem from which one could even get some interpretational advantages. In fact, from Eqs. (3.44) - (3.48) we notice that the violation of the dominant energy condition (DEC)

$$\rho + p = -V_{SQ}, \tag{4.9}$$

has an essentially quantum nature, so that such a violation vanishes in the classical limit where $\hbar \to 0$. In fact, it is currently believed that, even though classical general relativity cannot be accommodated to a violation of the dominant energy condition [66], such a violation can be admitted quantum mechanically, at least temporarily. Moreover, since the violating term $-V_{SQ}$ is directly related to the negentropy $H_{SQ} = -k_B$, it is really tempting to establish a link between that violation and the emergence of life in the universe. After all, one cannot forget that if living beings are fed on with negative entropy [64] then we ought to initially have some amount of negentropy to make the very emergence of life a more natural process which by itself satisfies the second law.

4.3 Holographic models

Holographic models which are related with the entropy of a dark energy universe have been extensively considered [21, 23, 24]. We shall discuss now the main equation that would govern the holographic model for the quantum cosmic scenario. If we try to adjust that model to the Li's holographic description for dark energy [24], then we had to define the holographic sub-quantum model by the relation

$$H^2 = \frac{8\pi G\rho}{3} = 4\pi G V_{SQ}\mu(t)^2 \ln\left(8GV_{SQ}R_h^2\right), \tag{4.10}$$

where the future event horizon $R_h = a(t)\int_t^\infty dt'/a(t')$ is given by

$$R_h = \frac{e^{x^2}}{\sqrt{8GV_{SQ}}}\left[1-\Phi(x)\right], \tag{4.11}$$

with $\Phi(x)$ the probability integral [54],

$$x = \frac{H_0}{\sqrt{8\pi GV_{SQ}}} + \sqrt{2\pi GV_{SQ}}t, \tag{4.12}$$

and

$$\mu(t)^2 = \frac{1}{1+3(1+w(t))\ln\left[1-\Phi\left(-\frac{1}{1+w(t)}\right)\right]}. \tag{4.13}$$

Note that: (1) $R_h \to \infty$ as $t\to\infty$ or $V_{SQ}\to 0$, (2) in the latter limit $H^2 \to 0$, (3) $\mu(t)^2$ is no longer a constant because we are dealing with a tracking model where the parameter w depends on time, and (4) the holographic model has no the problems posed by the usual holographic phantom energy models. However, this formulation does not satisfy the general holographic equation originally introduced by Li which reads [24] $\rho \propto H^2 \propto c^2/R^2$ (where R is the proper radius of the holographic surface and c is a parameter of order unity that depends on w according to the relation $w = -(1+2/c)/3$), and therefore seems not satisfactory enough. A better and quite simpler holographic description which comes from saturating the original bound on entropy [8] and conforms the general holographic equation stems directly from the very definitions of the energy density (3.44) and the entropy (4.7). Such a definition would read

$$\rho = \kappa S_{SQ}(a_+, H_0) = \frac{3H^2}{8\pi G} = \frac{3}{8\pi GR_H^2}. \tag{4.14}$$

It appears that if the last equality in Eq. (4.15) holds then the holographic screen is related to the Hubble horizon rather than the future event horizon or particle horizon. In order to confirm that identification we derive now the vacuum metric that can be associated to our ever-accelerating cosmic quantum model with the ansatz $ds^2 = -e^\nu dt^2 + e^\lambda dr^2 + r^2 d\Omega_2^2$. For an equation of state $p = w\rho$ the Einstein equations then are

$$e^{-\lambda}\left(\frac{\lambda'}{r} - \frac{1}{r^2}\right) + \frac{1}{2r^2} = 8\pi G\rho \tag{4.15}$$

$$e^{-\lambda}\left(\frac{\nu'}{r}+\frac{1}{r^2}\right)-\frac{1}{2r^2}=8\pi Gw\rho. \tag{4.16}$$

We get finally the non-static metric

$$ds^2=-\left(1-H^2r^2\right)^{-(1+3w)/2}dt^2+\frac{dr^2}{1-H^2r^2}+r^2d\Omega_2^2, \tag{4.17}$$

which consistently reduces to the de Sitter static metric for $w=-1$. It follows that there exists a time-dependent horizon at $r=H^{-1}$, which is always apparent for $w<-1/3$, playing in fact the role of a Hubble horizon, like in the de Sitter case. Thus, e.g. for the case that $w=-5/3$, the above metric reduces to

$$ds^2=\left(1-H^2r^2\right)^2dt^2+\frac{dr^2}{1-H^2r^2}+r^2d\Omega_2^2, \tag{4.18}$$

so that we can introduce a tortoise coordinate

$$r^*=\int\frac{dr}{(1-H^2r^2)^{3/2}}=-\frac{r}{\sqrt{1-H^2r^2}},\quad 0\leq r^*\leq\infty. \tag{4.19}$$

Using then advanced and retarded coordinates so that $U=t+r^*$ and $V=t-r^*$, we always can re-write the above metric as a line element which, in fact, is no longer singular at $r=H^{-1}$, i.e.

$$ds^2=-\left(1-H^2r^2\right)^2dUdV+r^2d\Omega_2^2, \tag{4.20}$$

in which r is implicitly determined from r^*. We can follow now the procedure described in [65] in order to obtain the maximally extended metric and from it the known expression for temperature and entropy of the de Sitter space.

This holographic model has several advantages over the previous Li's model [24] and other models [23], including its naturalness (it has been many times stressed that choosing the Hubble horizon is quite more natural than using, for the sake of mathematical consistency, particle or future event horizons), simplicity (no ad hoc assumption has been made), implication of an IR cutoff depending on time, formal equivalence with Barrow's hyper inflationary model [67] (but here respecting the thermodynamical second law as, in this case, $S_{SQ}(a_+,H_0)$ increases with time), and allowance of a unification between the present model and that for dark energy from vacuum entanglement [68].

4.4 Quantum cosmic models and entanglement entropy

The latter property deserves some further comments. In fact, if we interpret a^3V_{SQ} as the total entanglement energy of the universe, due to the additiviness of the entanglement entropy, one can then add up [68] the contributions from all existing individual fields in the observable universe, so that the entropy of entanglement $S_{\rm Ent} = \beta R_H^2$ (see comment after Eq. (4.8)), with β a constant including the spin degrees of freedom of quantum fields in the observable volume of radius R_H and a numerical constant of order unity. On the other hand, the presence of a boundary at the horizon leads us to infer that the entanglement energy ought to be proportional to the radius of the associated spherical volume, i.e. $E_{\rm Ent} = \alpha R_H$ [68], with α a given constant. We have then,

$$E_{\rm Ent} = a^3 V_{SQ} = \alpha R_H \tag{4.21}$$

$$S_{\rm Ent} = \beta R_H^2. \tag{4.22}$$

It is worth noticing that one can then interpret the used temperature as the entanglement temperature, so that $E_{\rm Ent} = k_B T(a_+)$. Now, integrating over R_H the expression for $dE_{\rm Ent}$ derived by Lee, Lee and Kim [68] from the saturated black hole energy bound [69],

$$dE_{\rm Ent} = T_{\rm Ent} dS_{\rm Ent} \tag{4.23}$$

(where $T_{\rm Ent} = (2\pi R_H)^{-1}$ is the Gibbons-Hawking temperature), we consistently recover expression (4.22) for $\alpha = \beta/\pi$. This result is also consistent with the holographic expression introduced before. It follows therefore that the quantum cosmic holographic model considered in the present paper can be consistently interpreted as an entangled dark energy holographic model, similar to the one discussed in [68], with the sub-quantum potential V_{SQ} playing the role of the entanglement energy density.

Before closing up this section, it would be worth mentioning that the recent data [70] seem to point to a value $w < -1$, with $\dot{w}$ small and positive, just the result predicted in the present chapter. We in fact note that from Eq. (3.45) we obtain that $\dot{w} = 4\dot{H}^2/(3H^3) \propto t^{-3}$, at sufficiently large time.

4.5 Conclusions

This chapter has investigated the thermodynamics of the two new four-dimensional cosmological models described in the previous chapter. While one of the models is ruled out on general thermodynamical grounds as being unphysical, the other model corresponds to an equation of state $p = w\rho$ with parameter $w < -1$, therefore evolving in the phantom region, but, as pointed out in Ch. 3, free from most of the problems associated with phantom scenarios: violent instabilities, future singularities and classical violations of energy conditions. We have shown furthermore that the considered phantom model implies a more consistent cosmic holographic description and the equivalence between the discussed models and the entangled dark energy model of the universe.

Indeed, if the ultimate cause for the current speeding-up of the universe is quantum entanglement associated with its matter and radiation contents, then one would expect that the very existence of the current universe would imply a violation of the Bell's inequalities and hence the quantum probabilistic description related to the sub-quantum potential considered in this work, or the collapse of the superposed cosmic quantum state into the universe we are able to observe, or its associated complementarity between cosmological and microscopic laws, any other aspects that may characterise a quantum system. The current dominance of quantum repulsion over attractive gravity started at a given coincidence time would then mark the onset of a new *quantum* region along the cosmic evolution, other than that prevailed at the Big Bang and early primeval universe, this time referring to the quite macroscopic, apparently classical, large universe which we live in. Thus, quite the contrary to what is usually believed, quantum physics does not just govern the microscopic aspects of nature but also the most macroscopic domain of it in such a way that we can say that current life is forming part and is a consequence of a true quantum system.

Chapter 5

Benigner phantom cosmology: a gracious exit from the matter-dominated phase

The occurrence of the scaling accelerated phase after matter dominance has been shown to be rather problematic for most existing dark energy and modified gravity models. In this chapter we consider a cosmic scenario where both the matter particles and scalar field are associated with sub-quantum potentials which make the effective mass associated with the matter particles to vanish at the coincidence time, so that a cosmic system where a matter dominance phase followed by an accelerating expansion is allowed. In this chapter we shall not use natural units, so $\hbar$ will appear explicitly in the equations. However,we shall take $c = 1$ but this will not lead to confusion.

5.1 Introduction

A recent paper by Amendola, Quartin, Tsujikawa and Waga (hereafter denoted as AQTW) [71] has put most existing models for dark energy in an apparent very serious trouble. Actually, if the result obtained by AQTW would be confirmed with full generality, then these authors have claimed that the whole paradigm of dark energy ought to be abandoned (see however the results in [72], e.g). Such as it happens with other aspects of the current accelerating cosmology, the problem is to some extend reminiscent of the difficulty initially confronted by earliest inflationary accelerating models [44] which could not smoothly connect with the following FRW decelerating evolution [73]. As is well known, such a difficulty was solved by invoking the new inflationary scenario [74]. In fact, the problem recently posed for dark energy can be formulated by saying that a previous decelerating matter-dominated era cannot be followed by an accelerating universe dominated by dark energy and it is in this sense that it can be somehow regarded as the time-reversed version of the early inflationary exit difficulty. In more technical terms what AQTW have shown is that it is impossible to find a sequence of matter and scaling acceleration for any scaling Lagrangian which can be approximated as a polynomial because a scaling Lagrangian is always singular in the phase space so that either the matter-dominated era is prevented or the region with a viable matter is isolated from that where the scaling acceleration occurs. Ways out from this problem required assuming either a sudden emergence of dark energy domination or a cyclic occurrence of dark energy, both assumptions being quite hard to explain and implement. In this chapter we however consider the dark energy model developed in Chs. 3 and 4, where such problems are no longer present due to some sort of quantum characteristics which can be assigned to the particles and radiation in that model.

5.2 On the onset of the cosmic accelerating phase

We start with an action integral that contains all the ingredients of our model. Such an action is a generalisation of the one used by AQTW which contains a time-dependent coupling between dark energy and matter and leads to a general Lagrangian that admits scaling solutions formally the same as those derived in [71]. Setting the Planck mass unity, our Lorentzian action reads

$$S = \int d^4x\sqrt{-g}\left[R + p(X,\phi)\right] + S_m\left[\psi_i, \xi, m_i(V_{SQ}), \phi, g_{\mu\nu}\right] + ST\left(K, \psi_i, \xi\right), \tag{5.1}$$

where g is the determinant of the four-metric, p is a generically non-canonical general Lagrangian for the dark-energy scalar field ϕ with kinetic term $X = g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$, formally the same as the one used in [71], S_m corresponds to the Lagrangian for the matter fields ψ_i, each with mass m_i, which is going to depend on a sub-quantum potential V_{SQ} in a way that will be made clear in what follows, so as on the time-dependent coupling ξ of the matter field to the dark energy field ϕ. The term ST denotes the surface term which generally depends on the trace on the second fundamental form K, the matter fields ψ_i and the time-dependent coupling $\xi(t)$ between ψ_i and ϕ for the following reasons.

We first of all point out that in the theory being considered the coupling between the matter and the scalar fields can generally be regarded to be equivalent to a coupling between the matter fields and gravity plus a set of potential energy terms for the matter fields. In fact, if we restrict ourselves to this kind of theories, a scalar field ϕ can always be mathematically expressed in terms of the scalar curvature R [75]. More precisely, for the scaling accelerating phase we shall consider a sub-quantum dark energy model (see [46] and [43, 45, 76]) in which, as we have already seen in Ch. 3, the Lagrangian for the field ϕ vanishes in the classical limit where the sub-quantum potential is made zero; i.e. we take $p = L = -V(\phi)\left(E(x,k) - \sqrt{1-\dot{\phi}^2}\right)$, where $V(\phi)$ is the potential energy and $E(x,k)$ is the elliptic integral of the second kind, with $x = \arcsin\sqrt{1-\dot{\phi}^2}$ and $k = \sqrt{1 - V_{SQ}^2/V(\phi)^2}$, and the overhead dot $\dot{}$ means derivative with respect to time. Using then a potential energy density for ϕ and the sub-quantum medium (note that the sub-quantum potential energy density becomes constant [43, 45, 76], see later on), we have for the energy density and pressure, $\rho \propto X(HV_{SQ}/\dot{H})^2 = p(X)/w(t)$, with $H \propto \phi V_{SQ} + H_0$, $\dot{H} \propto \sqrt{2X}V_{SQ}$, where H_0 is constant. For the resulting field theory to be finite, the condition that $2X = 1$ (i.e. $\phi = C_1 + t$) had to be satisfied [43, 45, 76], and from the Friedmann equation, the scale factor ought to be given by $a(t) \propto \exp\left(C_2 t + C_3 t^2\right)$, with C_1, C_2 and C_3 being constants. It follows then that for at least a flat space-time, we generally have $R \propto 1 + \alpha\phi^2$ (where α is another constant and we have re-scaled time) in that type of theories, and hence the matter fields - scalar field couplings, which can be generally taken to be proportional to $\phi^2\psi_i^2$, turn out to yield $\xi R\psi_i^2 - K_0\psi_i^2$, with K_0 again a given constant. The first term of this expression corresponds to a coupling between matter fields and gravity which requires an extra surface term, and the second one ought to be interpreted as a potential energy term for the matter fields $V_i \equiv V(\psi_i) \propto \psi_i^2$. In this way, for a general theory that satisfied the latter requirement, the action integral (5.1) should be re-written as

$$\begin{aligned} S = &\int d^4x\sqrt{-g}\left[R(1-\xi\psi_i^2) + p(X,\phi)\right] \\ &+ S_m\left[\psi_i, V_i, m_i(V_{SQ}), g_{\mu\nu}\right] - 2\int d^3x\sqrt{-h}\,\mathrm{Tr}K(1-\xi\psi_i^2), \end{aligned} \tag{5.2}$$

in which h is the determinant of the three-metric induced on the boundary surface and it can be noticed that the scalar field ϕ is no longer involved at the matter Lagrangian. We specialise now in the minisuperspace that corresponds to a flat FRW metric in conformal time $\eta = \int dt/a(t)$

$$ds^2 = -a(\eta)\left(-d\eta^2 + a(\eta)^2 d\mathrm{x}^2\right), \tag{5.3}$$

with $a(\eta)$ the scale factor. In this case, if we assume a time-dependence of the coupling such that it reached the value $\xi(\eta_c) = 1/6$ at the coincidence time η_c and choose suitable values for the arbitrary constants entering the above definition of R in terms of ϕ^2, then the action at that coincidence time would reduce to

$$S = \frac{1}{2}\int d\eta \left[a'^2 - \sum_i (\chi_i'^2 - \chi_i^2) + a^4\left(p(X,\phi) + \sum_i m_i (V_{SQ})^2\right)\right], \tag{5.4}$$

where the prime $'$ denotes derivative with respect to conformal time η and $X = \frac{1}{2a^2}(\phi')^2$. Clearly, the fields χ_i would then behave like though if they formed a collection of conformal radiation fields were it not by the presence of the nonzero mass terms m_i^2 also at the coincidence time. If for some physical cause the latter mass terms could all be made to vanish at the coincidence time, then all matter fields would behave like though they were a collection of radiation fields filling the universe at around the coincidence time and there would not be the disruption of the evolution from a matter-dominated era to a stable accelerated scaling solution of the kind pointed out by AQTW, but the system would smoothly enter the accelerated regime after a given brief interlude where the matter fields behave like pure radiation. In what follows we shall show that in the sub-quantum scenario considered above such a possibility can actually be implemented.

At the end of the day, any physical system always shows the actual quantum nature of its own. One of the most surprising implications taught by dark energy and phantom energy scenarios is that the universal system is not exception on that at any time or value of the scale factor. Thus, we shall look at the particles making up the matter fields in the universe as satisfying the Klein-Gordon wave equation [1] for a Bohmian quasi-classical wave function [46] $\Psi_i = R_i \exp(iS_i/\hbar)$, where we have restored an explicit Planck constant, R_i is the probability amplitude for the given particle to occupy a certain position within the whole homogeneous and isotropic space-time of the universe, as expressed in terms of relativistic coordinates, and S_i is the corresponding classical action also defined in terms of relativistic coordinates.

Taking the real part of the expression resulting from applying the Klein-Gordon equation to the wave function Ψ_i, and defining the classical energy as $E_i = \partial_i S/\partial t$ and the classical momentum as $p_i = \nabla S_i$, one can then derive the modified Hamilton-Jacobi equation

$$E_i^2 - p_i^2 + V_{SQi}^2 = m_{0i}^2, \tag{5.5}$$

where V_{SQi} is the relativistic version of the so-called sub-quantum potential [46] which is here given by

$$V_{SQi} = \hbar\sqrt{\frac{\nabla^2 R_i - \ddot{R}_i}{R_i}}, \tag{5.6}$$

[1] Strictly speaking, observable baryonic matter fields should be described by the Dirac equation rather than the Klein-Gordon equation. However, for our present purposes the use of the latter equation will suffice. On the other hand, even before coincidence time, most of the matter content had to be in the form of dark matter, a stuff whose nature is still unknown.

that should also satisfy the continuity equation (i.e. the probability conservation law) for the probability flux, $J = \hbar\, \mathrm{Im}(\Psi^*\nabla\Psi)/(m\mathrm{V})$ (with $\mathrm{V} \propto a^3$ the volume), stemming from the imaginary part of the expression that results by applying the Klein-Gordon equation to the wave equation Ψ. Thus, if the particles are assumed to move locally according to some causal laws [46], then the classical expressions for E_i and p_i will be locally satisfied. Therefore we can now interpret the cosmology resulting from the above formulae as a classical description with an extra sub-quantum potential, and average Eq. (5.5) with a probability weighting function for which we take $P_i = |R_i|^2$, so that

$$\int\int\int dx^3 P_i \left(E_i^2 - p_i^2 + V_{SQi}^2\right) = \langle E_i^2\rangle_{\rm av} - \langle p_i^2\rangle_{\rm av} + \langle V_{SQi}^2\rangle_{\rm av} = \langle m_{0i}^2\rangle_{\rm av}, \tag{5.7}$$

with the averaged quantities coinciding with the corresponding classical quantities and the averaged total sub-quantum potential squared being given by

$$\langle V_{SQ}^2\rangle_{\rm av} = \hbar^2\left(\langle\nabla^2 P\rangle_{\rm av} - \langle\ddot{P}\rangle_{\rm av}\right). \tag{5.8}$$

It is worth noticing that in the above scenario the velocity of the matter particles should be defined to be given by

$$\langle v_i\rangle_{\rm av} = \frac{\langle p_i^2\rangle_{\rm av}^{1/2}}{\left(\langle p_i^2\rangle_{\rm av} + \langle m_{0i}^2\rangle_{\rm av} - \langle V_{SQi}^2\rangle_{\rm av}\right)^{1/2}}. \tag{5.9}$$

It follows that in the presence of a sub-quantum potential, a particle with nonzero rest mass $m_{0i} \neq 0$ can behave like though if was a particle moving at the speed of light (i.e. a radiation massless particle) provided $\langle m_{0i}^2\rangle_{\rm av} = \langle V_{SQi}^2\rangle_{\rm av}$. Thus, if we introduce an effective particle rest mass $m_{0i}^{\rm eff} = \sqrt{\langle m_{0i}^2\rangle_{\rm av} - \langle V_{SQi}^2\rangle_{\rm av}}$, then we get that the speed of light again corresponds to a zero effective rest mass. It has been noticed [43, 45, 76], moreover, that in the cosmological context the averaged sub-quantum potential defined for all existing radiation in the universe should be regarded as the cosmic stuff expressible in terms of a scalar field ϕ that would actually make up our scaling dark-energy solution. At the coincidence time, that idea should actually extend in the present formalism to also encompass in an incoherent way, together with the averaged sub-quantum potential for CMB radiation, the averaged sub-quantum potential for matter particles, as a source of dark energy. On the other hand, it has been pointed out as well [43, 45, 76] that the sub-quantum potential ought to depend on the scale factor $a(t)$ in such a way that it steadily increases with time, being the sub-quantum energy density satisfying the above continuity equation what keeps constant along the whole cosmic evolution.

Assuming the mass m_i appearing in the action (5.4) to be an effective particle mass, it turns out that the onset of dark energy dominance would then be precisely at the coincidence time when $\langle V_{SQi}^2\rangle_{\rm av} \equiv \langle V_{SQi}(a)^2\rangle_{\rm av}$ reached a value which equals $\langle m_{0i}^2\rangle_{\rm av}$ and all the matter fields behaved in this way like a collection of radiation fields which are actually irrelevant to the issue of the incompatibility of the previous eras with a posterior stable accelerated current regime. In this case, the era of matter dominance can be smoothly followed by the current accelerated expansion where all matter fields would effectively behave like though if they cosmologically were tachyons. This interpretation would ultimately amount to the unification of dark matter and dark energy, as the dark energy model being dealt with here is nothing but a somehow quantised version of tachyon dark energy [77], so that one should expect both effective tachyon matter and tachyon dark energy to finally decay to dark matter, so providing a consistent solution to the cosmic coincidence problem.

Now, from our action integral (5.4) one can derive the equation of motion for the field ϕ; that is (see also [78] and [79])

$$\ddot{\phi}\left(p_X + 2Xp_{XX}\right) + 3Hp_X\dot{\phi} + 2Xp_{X\rho} - p_\phi = \frac{\delta S}{a^3\delta\phi}, \tag{5.10}$$

where we have restored the cosmic time t, using the notation of [71], [78] and [79], such that a suffix X or ϕ denotes a partial derivative with respect to X or ϕ, respectively, and now the last coupling term is time-dependent. Note that if we confine ourselves to the theory where $a(t)$ accelerates in an exponential fashion and $\dot{\phi}^2 = 1$ then the first term of this equation would vanish. Anyway, in terms of the energy density ρ for the scalar field ϕ the above general equation becomes formally the same as that which was derived in [71]

$$\frac{d\rho}{dN} + 3(1+w)\rho = -Q\rho_m\frac{d\phi}{dN}, \tag{5.11}$$

with ρ_m the energy density for the matter field, $N = \ln a$, and $Q = -\frac{1}{a^3\rho_m}\frac{\delta S_m}{\delta\phi}$. We can then derive the condition for the existence of scaling solutions for time-dependent coupling which, as generally the latter two equations are formally identical to those derived by AQTW, is the same as that was obtained by these authors. Hence, we have the generalised master equation for p [71]

$$\left[1 + \frac{2dQ(\phi)}{\lambda Q^2 d\phi}\right]\frac{\partial \ln p}{\partial \ln X} - \frac{\partial \ln p}{\lambda Q\partial\phi} = 1, \tag{5.12}$$

whose solution was already obtained by AQTW [71] to be:

$$p(X,\phi) = XQ(\phi)^2 g\left(XQ(\phi)^2 e^{\lambda\kappa(\phi)}\right) \tag{5.13}$$

where g is an arbitrary function, λ is a given function of the parameters of the equations of state for matter and ϕ and the energy density for ϕ, being $\kappa = \int^\phi Q(\xi)d\xi$ (see [71]). In the phase space we then have an equation-of-state effective parameter for the system $w_{\rm eff} = -1 - \frac{2\dot{H}}{3H^2} = gx^2 + z^2/3$, with H the Hubble parameter and x and z respectively being $x = \dot{\phi}/(\sqrt{6}H)$ and $z = \sqrt{\rho_{\rm rad}/(3H^2)}$. At the coincidence time where we have just radiation ($z \neq 0$ and $\rho_m = \rho_{\rm rad}$) the effective equation of state is [71] $w_{\rm eff} = 1/3$. Hence at the coincidence time interval we can only have radiation, neither matter or accelerated expansion domination, just the unique condition that would allow the subsequent onset of the accelerated expansion era where conformal invariance of the field χ no longer holds.

Thus, it appears that in the considered model a previous matter-dominated phase can be evolved first into a radiation phase at a physical regular coincidence short stage which is then destroyed to be finally followed by the required new, independent phase of current accelerating expansion. This conclusion can be more directly drawn if one notices that there is no way by which the general form of the Lagrangian (5.13) can accommodate the Lagrangian final form $L \equiv p = f(a, \dot{a})\dot{\phi}^2 V_{SQ}^2$ which characterizes sub-quantum dark energy models whose pressure p vanishes in the limit $V_{SQ} \to 0$. It thus appears that at least these models can be taken to be counter examples to the general conclusion that current dark-energy and modified gravity models (see however [72]) are incompatible with the existence of a previous matter-dominated phase, as suggested in [71]. We finally notice, moreover, that the kind of sub-quantum dark energy theory providing the above counter example is one which shows no classical analog (i.e. the Lagrangian, energy density and pressure are all zero in the classical limit $\hbar \to 0$) and is thereby most economical of all. Thus, the above conclusion can also be stated by saying that, classically, a previous phase of matter dominance is always compatible with the ulterior emergence of a dominating phase made up of "nothing". In this way, similarly to as the abrupt, unphysical exit of the old inflationary problem was circumvented by introducing [74] a scalar field potential with a flat plateau leading to a "slow-rollover" phase transition, the abrupt disruption of the scaling phase after matter dominance can be also avoided by simply considering a vanishing scalar field potential that smooths the transition and ultimately makes it to work.

5.3 Conclusions

In order to solve the coincidence problem, a cosmological model should show that dark energy and dark matter follow the same scaling solution from some time onward. Likewise, the model should contain a sufficiently long matter dominated epoch to permit the structure formation observed. This implies that a sequence of epochs: radiation, matter-dominated and an accelerating scaling attractor must take place. According to [71], this is impossible for any scaling Lagrangian which can be approximated as a polynomial with both positive and negative integer powers of its argument. In this chapter we have shown that the benigner phantom model explored in the previous chapter, contrary to what is claimed in

[71], allows the occurrence of a matter-dominated era followed by an accelerated expansion.

Chapter 6

Multidimensional quantum cosmic models: generalised solutions and gravitational waves

This chapter contains a discussion on the quantum cosmic models, starting with the interpretation that all of the accelerating effects in the current universe are originated from the existence of a nonzero entropy of entanglement. In such a realm, we obtain new cosmic solutions for any arbitrary number of spatial dimensions, studying the stability of these solutions, so as the emergence of gravitational waves in the realm of the most general models.

6.1 Generalised cosmic solutions

It has been already seen that the quantum cosmic solutions can be regarded as either some generalisations from the flat version of the de Sitter space or, if V_{SQ} is sufficiently small, such as it appears to actually be the case, as perturbations of that de Sitter space. Since most of such models correspond to equations of state whose parameter is less than -1, such as it was mentioned before, they are also known as *benigner* phantom cosmic models. In this section we shall derive even more general expressions for these quantum cosmic solutions by (i) considering the similar generalisations or perturbations of the hyperbolic version of the de Sitter space, and (ii) using a d-dimensional manifold. Actually, some observational data have implied that our universe is not perfectly flat and recent works [80, 81] contemplate the possibility of the universe having spatial curvature. Thus, although WMAP alone abhors open models, requiring $\Omega_{total} \equiv \Omega_m + \Omega_\Lambda = 1 - \Omega_k \geq 0.9$ (95%), closed model with Ω_{total} as large as 1.4 are still marginally allowed provided that the Hubble parameter $h \sim 0.3$ and the age of the Universe $t_0 \sim 20Gyr$. The combinations of the WMAP plus the SNIa data or the Hubble constant data also imply the possibility of the closed universe, giving curvature parameters $k = -0.011 \pm 0.012$ and $k = -0.014 \pm 0.017$, respectively [80], although the estimated values are still consistent with the flat FRW world model. Moreover, in [82] it is said that the best fit closed universe model has $\Omega_m = 0.415$, $\Omega_\Lambda = 0.630$ and $H_0 = 55kms^{-1}Mpc^{-1}$ and is a better fit to the WMAP data alone than the flat universe model ($\Delta\chi^2_{eff} = 2$). However, the combination of WMAP data with either SNe data, large-scale structure data or measurements of H_0 favours models with Ω_K close to 0.

The d-dimensional de Sitter space has already been considered elsewhere [83]. Here we shall extend it to the also maximally symmetric space whose spacetime curvature is still negative (positive Ricci scalar) but no longer constant. Our spacetime will be a solution of Einstein equation

$$R_{ab} = t_{ab}, \quad a, b = 0, 1, ...(d-1), \tag{6.1}$$

with

$$t_{ab} = (H \pm \hbar\xi t)^2 g_{ab}, \tag{6.2}$$

where $H^2 = \Lambda/(d-1)$ is a cosmological constant and the constant $\hbar\xi$ generalises the sub-quantum potential considered in the quantum cosmic models described in the previous chapters. We notice that in the classical limit $\hbar \to 0$ the above definition becomes that of the usual d-dimensional de Sitter space. We shall restrict ourselves in this paper to the case in which our generalised d-dimensional de Sitter space can still be visualised as a $d+1$ hyperboloid defined as [84]

$$-x_0^2 + \sum_{j=1}^{d} x_j^2 = H^{-2}. \tag{6.3}$$

This hyperboloid is embedded in E^{d+1}, so that the most general expression of the metric for our extended quantum-corrected solutions is provided by the metric induced in this embedding, that is

$$ds^2 = -dx_0^2 + \sum_{j=1}^{d} dx_j^2, \tag{6.4}$$

which has the same topology and invariance group as the d-dimensional de Sitter space [83].

This metric can now be exhibited in coordinates $\Theta_\pm = t(1 \pm \hbar\xi t/H)\epsilon(\mp H_0/(4\hbar\xi), \pm\infty)$ (notice that our solutions only then cover a portion of the de Sitter time, while $t\epsilon(-\infty, +\infty)$),

$\psi_{d-1}, \psi_{d-2}, ...\psi_2 \epsilon (0, \pi)$, $\psi_1 \epsilon (0, 2\pi)$, defined by

$$x_d = H^{-1} \cosh(H\Theta) \sin\psi_{d-1} \sin\psi_{d-2} ... \sin\psi_2 \cos\psi_1$$

$$x_{d-1} = H^{-1} \cosh(H\Theta) \sin\psi_{d-1} \sin\psi_{d-2} ... \sin\psi_2 \sin\psi_1$$

$$x_{d-2} = H^{-1} \cosh(H\Theta) \sin\psi_{d-1} \sin\psi_{d-2} ... \cos\psi_2 \tag{6.5}$$

$$x_1 = H^{-1} \cosh(H\Theta) \cos\psi_{d-1}$$

$$x_0 = H^{-1} \sinh(H\Theta),$$

which should be referred to as either time Θ_+ or time Θ_-. In terms of these coordinates metric (6.4) splits into

$$ds_\pm^2 = -\left(1 \pm \frac{2\hbar\xi t}{H}\right)^2 dt^2$$
$$+H^{-2} \cosh^2 \left[t \left(H \pm \hbar\xi t\right)\right] d\Omega_{d-1}^2, \tag{6.6}$$

where $d\Omega_{d-1}^2$ is the metric on the $(d-1)$-sphere. Metric (6.10) is a closed $(d-1)$-dimensional FRW metric whose spatial sections are $(d-1)$-spheres of radius $H^{-1}\cosh(H\Theta)$. The coordinates defined by Eqs. (6.5) describe two closed quantum cosmic spaces, $B_\pm$, which interconvert into each other at $t = 0$. B_+ first steadily contracts until $t = 0$ where it converts into B_- to first expand up to a finite local maximum value at $t = H/(2\hbar\xi)$, then contract down to a_0 at $t = H/(\hbar\xi)$, expanding thereafter to infinite. B_- would first contract until $t = -H/(\hbar\xi)$, then expand up to reach a local maximum at $t = -H/(2\hbar\xi)$, to contract again until $t = 0$, where it converts into a_+ which will steadily expand thereafter to infinite (see Fig. 6.1 where the scale factor for these spaces is compared with that of the de Sitter space).

In terms of the conformal times $\eta_\pm = \int d\Theta_\pm / a_\pm$, which is given by

$$\tan\eta_\pm = \sinh\left(t \pm \hbar\xi t^2/H\right), \tag{6.7}$$

with $\pi/2 \geq \eta_+ \geq 0$ and $3\pi/2 \geq \eta_- \geq \pi$, the metrics can be re-expressed in a unitary form as

$$ds_\pm^2 = \frac{a_0^2}{\cos^2\eta_\pm}\left(-d\eta_\pm^2 + \gamma_{\alpha\beta} dx^\alpha dx^\beta\right), \alpha,\ \beta = 1, 2, ...(d-1), \tag{6.8}$$

where $\gamma_{\alpha\beta}$ is the metric for a unit $(d-1)$-sphere.

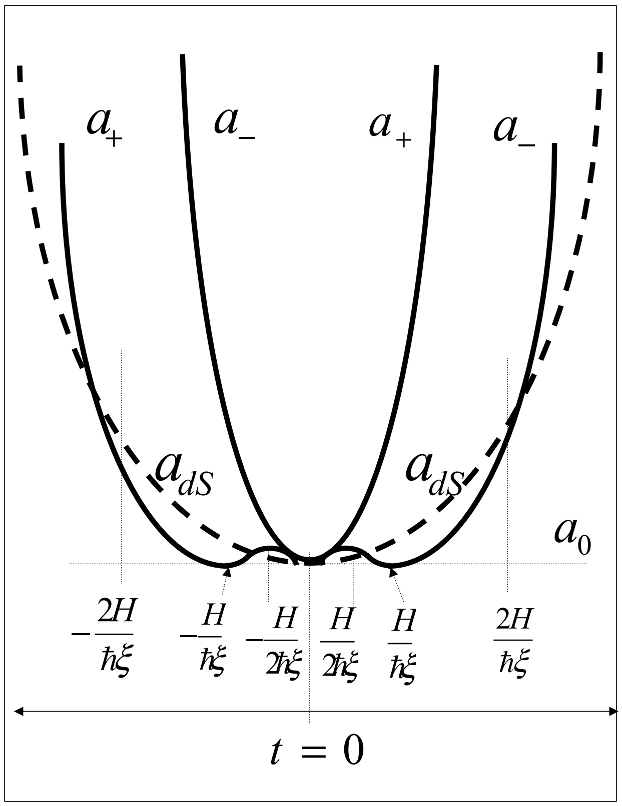

Figure 6.1: Cosmic solutions in the generalised quantum cosmic models, as compared with the de Sitter space. Solution a_+ for $t > 0$ and solution a_- for $t < 0$ go like in de Sitter space with the same H_0, but with higher acceleration. Solution a_+ for $t < 0$ and solution a_- for $t > 0$ correspond to a universe which is initially expanding in a decelerating way, then contract to a_0 and finally expands in an accelerated way towards infinite as $t \to \infty$.

We shall consider in what follows the equivalent in our quantum cosmic scenarios of the static $(d-1)$-dimensional metric. Using the new coordinates

$$x_d = H^{-1} \sin\psi_{d-1} \sin\psi_{d-2} ... \sin\psi_2 \cos\psi_1$$

$$x_{d-1} = H^{-1} \sin\psi_{d-1} \sin\psi_{d-2} ... \sin\psi_2 \sin\psi_1$$

$$x_{d-2} = H^{-1} \sin\psi_{d-1} \sin\psi_{d-2} ... \cos\psi_2$$

$$\tag{6.9}$$

$$x_3 = H^{-1} \sin\psi_{d-1} \sin\psi_{d-2} cos\psi_{d-3}$$

$$x_2 = H^{-1} \sin\psi_{d-1} \cos\psi_{d-2}$$

$$x_0 = H^{-1} \cos\psi_{d-1} \sinh(H\Theta')$$

where the coordinates are defined by $t'\epsilon(-\infty, +\infty))$, $r\epsilon(0, H^{-1})$, $\psi_{d-1}, \psi_{d-2}, ...\psi_2\epsilon(0, \pi)$, $\psi_1\epsilon(0, 2\pi)$. These coordinates will again be referred to either time Θ'_+ or time Θ'_-. Setting $r = H^{-1}\sin\psi_{d-1}$, we then find the metrics

$$ds_\pm^2 = -\left(1 \pm \frac{\hbar\xi t'}{H}\right)^2 dt'^2 \left(1 - H^2 r^2\right) + \frac{dr^2}{1 - H^2 r^2} + r^2 d\Omega_{d-2}^2, \tag{6.10}$$

where $d\Omega_{d-2}^2$ is the metric on the $(d-2)$-sphere. We immediately note that this metric is no longer static. The coordinates defined by that metric cover only the portion of the spaces with $x_1 > 0$ and $\sum_{j=2}^d x_j^2 < H^{-2}$, i.e. the region inside the particle and event horizons of an observer moving along $r = 0$.

Respective instantons can now be obtained by analytically continuing $\Theta_\pm \to i\mathrm{T}_\pm$ (where we have taken $\Theta' \equiv \Theta$ for the sake of simplicity in the expressions), that is $t' \to i\tau$ and $\xi \to -i\chi$, which contain singularities at $r = H^{-1}$, which are only apparent singularities if $\mathrm{T}_\pm$ are identified with periods $\pm 2\pi H^{-1}$, or in other words, if τ is respectively identified with periods $H(\sqrt{1 + 8\pi\hbar\chi H^{-2}} \pm 1)/(2\hbar\chi)$. It follows then that the two spaces under consideration would respectively behave as though if they would emit a bath of thermal radiation at the intrinsic temperatures given by

$$T_\pm^{th} = \frac{2\hbar\chi}{H\left(\sqrt{1 + 8\pi\hbar\chi H^{-2}} \pm 1\right)}. \tag{6.11}$$

It must be remarked that in the limit when $\chi \to 0$, both temperatures $T_\pm^{th}$ consistently reduce to the unique value $H/(2\pi) = \sqrt{\Lambda(d-1)^{-1}}/(2\pi)$, that is the temperature of a d-dimensional de Sitter space [83], even though T_-^{th} does it more rapidly than T_+^{th} (in fact, for sufficiently small χ, we can check that $T_-^{th} \simeq H/(2\pi)$ and $T_+^{th} \simeq \hbar\chi H/(H^2 + 2\pi\hbar\chi)$). Note that while we keep $\hbar$ in all definitions concerning the quantum cosmic spaces, natural units so that $\hbar = G = c = k_B = 1$ are otherwise used when such definitions are used. Now, one can estimate the entropy of these spaces by taking the inverse to their temperature. Thus, it can be seen that the entropy of the universe with scale factor a_+ will always be larger than that for a universe with scale factor a_-. It follows then that whereas the transition from a_+ to a_- at $t = 0$ would violate the second law of thermodynamics, the transition from a_- to a_+ at $t = 0$ would satisfy it (see Fig. 6.1), so making the model with scale factor a_+ evolving along positive time more likely to happen.

The time variables t and t' in Eqns. (6.2), (6.5) and (6.9) do not admit any bounds other than $(-\infty, +\infty)$, so that the involved models can be related with the Barrow's hyper inflationary model [67], albeit the solution a_+ here always respect the second law of thermodynamics because for such a solution the entropy is an ever increasing function of time [76].

Before closing up this section we shall briefly consider the static Schwarzschild-quantum mechanically perturbed solutions. It can be shown that in that case the line element is again not properly static as they depend on time in their g_{tt} component, that is

$$ds_\pm^2 = -\left(1 \pm \frac{\hbar\xi t'}{H}\right)^2 dt'^2 \left(1 - \frac{2M}{r} - H^2 r^2\right) + \frac{dr^2}{1 - \frac{2M}{r} - H^2 r^2} + r^2 d\Omega_{d-2}^2, \tag{6.12}$$

Instantons for such solutions can also be similarly constructed. One readily may show that again such instantons describe thermal baths at given temperatures expressed now by

$$T_\pm^{th} = \frac{2\hbar\chi}{H\left(\sqrt{1 + 8\pi\hbar\chi H^{-2}} \pm 1\right)\left(1 \mp \frac{2}{3}\epsilon\right) + O\left(\epsilon^2\right)}, \tag{6.13}$$

where the second sign ambiguity in the denominator refers to the cosmological (upper) and black hole (lower) horizons and, according to Ginsparg and Perry [85], $9M^2\Lambda = 1 - 3\epsilon^2$, with $0 \leq \epsilon << 1$, the degenerate case corresponding just to $\epsilon \to 0$.

6.2 Gravitational waves and semiclassical instability

In this section we shall restrict ourselves to the solutions derived in the previous section for just the four-dimensional case, considering the generation of gravitational waves in the realm of such solutions and some semiclassical instabilities that arise when one Euclideanises $(t \to i\tau)$ the higher-dimensional solutions. Thus, let us consider first the tensorial Liftshif-Khalatnikov perturbations corresponding to the zeroth mode $\ell = 0$. From them we can derive [83, 85] the differential equation

$$\nu'' + 2\tan\eta\nu' = 0, \tag{6.14}$$

where η and $' = d/d\eta$ refer to the conformal time, either η_+ or η_-, defined in Eq. (6.7). This differential equation has as general solution

$$\nu = C_0 + C_1\left(\eta + \frac{1}{2}\sin(2\eta)\right), \tag{6.15}$$

where C_0 and C_1 are given integration constants. We must now particularise solution (6.15) to be referred to $\eta_\pm$. In the case η_+ we see that the conformal time runs from 0 ($t = 0$) to $\pi/2$ ($t = \infty$). These waves do not destabilise the space as, though their amplitude does not vanish at the limit where $\eta_+ \to \pi/2$, neither it grows with time t. For η_- the conformal time runs from π ($t = 0$ or $t = H^2/(\hbar\xi)$) to $3\pi/2$ ($t = \infty$). It can be easily seen that neither these waves can destabilise the space.

For the general case $\ell \neq 0$, we have the general differential equation, likewise referred to either η_+ or η_-,

$$\nu'' + 2\tan\eta\nu' + \ell(\ell+2)\nu = 0. \tag{6.16}$$

The solution to this differential equation can be expressed as

$$\nu = \cos^3\eta C^{(2)}_{\ell-1}(\sin\eta), \tag{6.17}$$

with $C^{(\alpha)}_n$ the ultraspherical (Gegenbauer) polynomials of degree 2. Now, for $\eta_+ = 0$ or $\eta_- = \pi$, the amplitude vanishes for even $\ell = 2, 4, 6, \ldots$, and becomes

$$\nu = (-1)^{(\ell-1)/2}\frac{\Gamma\left(2 + \frac{\ell-1}{2}\right)}{\Gamma(2)\left(\frac{\ell-1}{2}\right)!},$$

for odd $\ell = 1, 3, 5, \ldots$. For $\eta_+ = \pi/2$, $\nu = (\ell+2)!/[6(l-1)!]$ and for $\eta_- = 3\pi/2$, $\nu = (-1)^{\ell-1}(\ell+2)!/[6(l-1)!]$. Once again the considered spaces are therefore stable to tensorial perturbations for nonzero ℓ. It is worth mentioning that for the solution corresponding to η_- and even ℓ, the absolute value of the amplitude of the gravitational waves would first increase from zero (at $t = 0$) to reach a maximum value at $t = H/(2\hbar\xi)$, to then decrease down to zero at $t = H/(\hbar\xi)$, and finally steadily increase all the time to reach its final finite value of unit order as $t \to \infty$. Clearly. a distinctive observational effect predicted by that cosmic model would be the generation of gravitational waves whose amplitude adjusted to the given pattern.

A general derivation of Eqns. (6.14) and (6.16) from a general traceless rank-two tensor harmonics which is an eigenfunction of the Laplace operator on S^3 and satisfies the eigenvalue equation $\nabla_a\nabla^a H^{(n)}_{cd} = -(n^2 - 3)H^{(n)}_{cd}$ can be found in [83, 85].

We add finally some comments to the possibility that our closed spaces develop a semi-classical instability. We shall use the Euclidean approach. In order to see if our Euclideanised solutions are stable or correspond to semiclassical instabilities, it will suffice to determine the eigenvalues of the differential operator [85, 86]

$$G_{abcd}\phi^{ab} \simeq -\Box\phi_{cd} - 2R_{acbd}\phi^{ab} \simeq \lambda\phi_{cd}, \tag{6.18}$$

where ϕ_{ab} is a metric perturbation. Now, if all $\lambda \geq 0$, the Euclideanised spaces are stable, showing a semiclassical instability otherwise. Stability can most readily be shown if, by analytically continuing metrics (6.10), the metric on the $(d-2)$-sphere, $d\Omega_{d-2}$, turns out to be expressible as the Kahler metric associated to a 2-sphere. Thus, let us introduce the complex transformation

$$Z = 2\tan(\psi_{d-2}/2)\exp\left(i\int d\Omega_{d-3}\right), \tag{6.19}$$

and hence in fact we can derive

$$d\Omega_{d-2} = \frac{d\bar{Z}dZ}{\left(1+\frac{1}{4}\bar{Z}Z\right)^2} \tag{6.20}$$

and the Kahler potential

$$K = 2\log\left(1+\frac{1}{4}\bar{Z}Z\right), \tag{6.21}$$

so showing that, quite similarly to what it happens in the d-dimensional de Sitter space, the instantons constructed from metrics (6.10) are stable. Whether or not a space-time corresponding to Schwarzschild-generalised de Sitter metric would show a semiclassical instability is a question that would require further developments and calculations.

6.3 Conclusions

This chapter has dealt with new four-dimensional and d-dimensional cosmological models describing an accelerating universe in the spatially flat and closed cases. The ingredients used for constructing these solutions are minimal as they only specify a cosmic relativistic field described by just Hilbert-Einstein gravity and the notion of the quantum entanglement of the universe, that is the probabilistic quantum effects associated with the general matter content existing in the universe or its generalisation for the closed cases. Two of such models correspond to an equation of state $p = w\rho$ with parameter $w < -1$ for their entire evolution, and still other of them which covers a period in its future also with $w < -1$;

that is to say, these three solutions are associated with the so-called phantom sector, showing however a future evolution of the universe which is free from most of the problems confronted by usual phantom scenarios; namely, violent instabilities, future singularities, incompatibility with the previous existence of a matter-dominated phase, classical violations of energy conditions or inadequacy of the holographic description. Therefore we also denote such quantum cosmic models as *benigner phantom* models. All these models can be regarded as generalisations or perturbations of the either exponential or hyperbolic form of the de Sitter space. The hyperbolic solution are given in a d-dimensional manifold which is particularised in the four-dimensional case in the Euclideanised extension that allowed us to derive quantum formulas for the temperature that reduce to that of Gibbons-Hawking when the perturbation is made to vanish. Finally, the generation of gravitational waves in some of the considered models has been studied in the realm of the Lifshitz-Khalatnikov perturbation formalism for the spatially closed case. It is also shown that none of these waves destabilise the space-time, as neither the vector and scalar cosmological perturbations do in the spatially flat and closed cases.

Chapter 7

Holographic dark energy: kinetic k-essence and dilatonic models

In this chapter, we shall use the kinetic k-essence and the dilaton scalar fields as effective descriptions of the underlying theory of dark energy, which we assume to be the holographic dark energy. We shall consider a connection between the holographic dark energy density and the kinetic k-essence and dilaton energy densities, respectively, in a flat Friedmann-Robertson-Walker universe. With the choice $c \geq 1$, the holographic dark energy can be described by a kinetic k-essence scalar field as well as by the dilaton scalar field. We reconstruct their kinetic terms as well as the dynamics of these holographic models and show that these models can describe the observed accelerated expansion of our universe with the choice $c \geq 1$.

7.1 Introduction

We have seen already in Sec. 2.3.1 that taking the future event horizon as the IR cut-off allows the construction of a viable HDE model. As a matter of fact, a time varying dark energy gives a better fit than a cosmological constant according to some analysis of astronomical data coming from type Ia supernovae [87]. However, it must be stressed that almost all dynamical dark energy models are settled at the phenomenological level and the HDE model is no exception in this respect. Its advantage, when compared to other dynamical dark energy models, is that the HDE model originates from a fundamental principle in quantum gravity [8,16], and therefore possesses some features of an underlying theory of dark energy. It is then fair to claim that the simplicity and reasonable nature of HDE provide a more reliable framework for investigating the problem of dark energy compared with other models proposed in the literature. For instance, the coincidence problem is substantially alleviated in some models of HDE based on the assumption that dark matter and HDE interact, with a decay of HDE into dark matter. This will be explored in the next chapter for the case of the tachyon field.

On the other hand, as is well known, scalar field models are an effective description of an underlying theory of dark energy. They are popular not only because of their mathematical simplicity and phenomenological richness, but also because they naturally arise in particle physics including supersymmetric field theories and string/M theory [1]. However, these fundamental theories do not predict their potential $V(\phi)$ or kinetic term uniquely. We are interested in the following: if we assume the HDE scenario as the underlying theory of dark energy, how a scalar field model can be used to effectively describe it. Therefore, it is meaningful to reconstruct the $V(\phi)$ or kinetic term kinetic term of a dark energy model possessing some significant features of the quantum gravity theory, such as the HDE model. In order to do that, the procedure is to establish a correspondence between the scalar field and the HDE by identifying their respective energy densities and then reconstruct the potential (if the scalar field is quintessence or the tachyon, for instance) or the kinetic term (k-essence or the dilaton belong to this class) and the dynamics of the field. In this chapter, within the different candidates to play the role of the dark energy, we have chosen the kinetic k-essence and the dilaton (when this behaves as a scalar field), as these have emerged as possible sources of dark energy [91–93]. Some work has already been done in this direction. Holographic quintessence and holographic quintom models have been discussed in [94] and [95], respectively, and the holographic tachyon model in [96]. Other relevant works can be found in [97]. As stated above, the aim in this chapter is to construct the holographic kinetic k-essence and dilatonic models of dark energy, relating the kinetic k-essence and dilaton scalar fields with the HDE.

7.2 K-essence

As a dark energy candidate, k-essence [91,92,98] is usually defined as a scalar field ϕ with a non-canonical kinetic energy associated with a lagrangian $\mathcal{L} = -V(\phi)F(X)$, where V is the potential and $X = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi$ is the kinetic term. In the case of the k-essence scalar field, the negative pressure that explains the accelerated expansion arises out of modifications to

[1] to see, for instance, how quintessence and tachyon models arise quite naturally out of the framework of string theory, consult [88] and [89, 90], respectively

the kinetic energy.

K-essence models are described by an effective minimally coupled scalar field with a non-canonical term. If for a moment we neglect the part of the Lagrangian containing ordinary matter, the general action for a k-essence field ϕ minimally coupled to gravity is

$$S = S_G + S_\phi = -\int d^4x\sqrt{-g}\left(\frac{R}{2} + F(\phi, X)\right), \tag{7.1}$$

where $F_k(\phi, X)$ is an arbitrary function of ϕ and its kinetic energy X. A possible motivation for actions of this form comes from considering low-energy effective string theory in the presence of a high-order derivative terms.

In what follows, we shall restrict ourselves to the simple k-essence models for which the potential $V = V_0 =$ constant [99]. We also assume that $V_0 = 1$ without any loss of generality. One reason for studying k-essence it that it is possible to construct a particularly interesting class of such models in which the k-essence energy density tracks the radiation energy density during the radiation-dominated era, but then evolves toward a constant-density dark energy component during the matter-dominated era. Such behaviour can to a certain degree solve the coincidence problem [91,92,98]. Because of this dynamical attractor behaviour, the cosmic evolution is insensitive to initial conditions. Another feature of k-essence is that it changes its speed of evolution in dynamic response to changes in the background equation of state.

7.2.1 Kinetic k-essence

We now restrict ourselves to the subclass of kinetic k-essence, with an action independent of ϕ

$$S = -\int d^4x\sqrt{-g}F(X). \tag{7.2}$$

The consideration of constraints on purely kinetic k-essence models from the latest observational data by applying model comparison statistics (F-test, AIC_c, and BIC) has found that these models are favoured over the ΛCDM by the combined data [100].

We assume a FRW metric $ds^2 = dt^2 - a^2(t)\, d\vec{x}^2$ (where $a(t)$ is the scale factor). Unless stated otherwise, we consider ϕ to be smooth on scales of interest so that $X = \frac{1}{2}\dot{\phi}^2 \geq 0$. The energy-momentum tensor of the k-essence is obtained by varying the action (7.2) with respect to the metric, yielding

$$T_{\mu\nu} = F_X \partial_\mu\phi\partial^\mu\phi - g_{\mu\nu}F, \tag{7.3}$$

where the subscript X denotes differentiation with respect to X. Identifying (7.3) with the energy-momentum tensor of a perfect fluid we have the k-essence energy density ρ_ϕ and pressure p_ϕ

$$\rho_\phi = F - 2XF_X \tag{7.4}$$

and

$$p_\phi = -F. \tag{7.5}$$

Throughout this chapter, we will assume that the energy density is positive so that $F - 2XF_X > 0$. The equation of state for the k-essence fluid can be written as $p_\phi = w_\phi \rho_\phi$ with with $F > 0$,

$$w_\phi = \frac{p_\phi}{\rho_\phi} = \frac{F}{2XF_X - F}. \tag{7.6}$$

As long as the condition $|2XF_X| \ll |F|$ is satisfied, w_ϕ can be close to -1.

On the other hand, the effective sound speed is given by

$$c_s^2 = \frac{\partial p_\phi/\partial X}{\partial \rho_\phi/\partial X} = \frac{F_X}{F_X + 2XF_{XX}} = \frac{F_X^2}{(XF_X^2)_X}, \tag{7.7}$$

where $F_{XX} \equiv d^2F/dX^2$. The definition of the sound speed comes from the equation describing the evolution of linear adiabatic perturbations in a k-essence dominated universe [101] (the non-adiabatic perturbation was discussed in [102], here we only consider the case of adiabatic perturbations). Perturbations can become unstable if the sound speed is imaginary, $c_s^2 < 0$, so we insist on $c_s^2 > 0$, or equivalently, $(XF_X^2)_X > 0$. Another potentially interesting requirement to consider is $c_s^2 \leq 1$, which says that the sound speed should not exceed the speed of light, which suggests violation of causality. Though this is an open problem (see e. g. [103–108]), we still impose this condition. It is important to notice that the k-essence models constructed to solve the coincidence problem inevitably give rise to the superluminal propagation of the field ($c_s^2 > 1$) at some stage of the cosmological evolution [106].

For a flat FRW metric, applying the Euler-Lagrange equation for the field to the action (7.2) we find the equation of motion for k-essence field

$$(F_X + 2XF_{XX})\ddot{\phi} + 3HF_X\dot{\phi} = 0, \tag{7.8}$$

which can be rewritten in terms of X as

$$(F_X + 2XF_{XX})\dot{X} + 6HF_X X = 0, \tag{7.9}$$

where the dot denotes differentiation with respect to the cosmic time and $H = \dot{a}/a$ is the Hubble parameter. If we now change the independent variable from time t to the scale factor a, we obtain

$$(F_X + 2XF_{XX})\, a \frac{dX}{da} + 6F_X X = 0. \tag{7.10}$$

This equation can be integrated exactly, for arbitrary F, yielding

$$XF_X^2 = ka^{-6}, \tag{7.11}$$

where k is a constant of integration [99]. Given a function $F(X)$, Eq.(7.11) allows us to find solutions $X(a)$ and then the other parameters of the k-essence fluid like ρ_ϕ, p_ϕ, ω_ϕ and c_s^2 as a function of the scale factor, a.

7.3 Holographic kinetic k-essence model

In order to build our holographic model, we impose the holographic nature to the kinetic k-essence, i.e., we identify ρ_ϕ with ρ_Λ.

We consider a universe filled with a matter component ρ_m (including both baryons and cold dark matter) and an holographic kinetic k-essence component ρ_ϕ, the Friedman equation reads

$$3M_P^2 H^2 = \rho_m + \rho_\phi, \tag{7.12}$$

or equivalently

$$H(z) = H_0 \left(\frac{\Omega_{m0}(1+z)^3}{1-\Omega_\phi} \right)^{1/2} \tag{7.13}$$

where $z = (1/a) - 1$ is the redshift of the universe. From the definition of the HDE and the definition of the future event horizon, we find

$$\int_a^\infty \frac{da'}{Ha'^2} = \int_x^\infty \frac{dx}{Ha} = \frac{c}{\sqrt{\Omega_\phi} Ha} \tag{7.14}$$

The Friedman equation (7.13) implies

$$\frac{1}{Ha} = \sqrt{a(1-\Omega_\phi)} \frac{1}{H_0\sqrt{\Omega_{m0}}} \tag{7.15}$$

Substituting (7.15) into (7.14), we obtain the following equation

$$\int_{x}^{\infty} e^{x'/2}\sqrt{1-\Omega_{\phi}}dx' = ce^{x/2}\sqrt{\frac{1}{\Omega_{\phi}}-1}, \tag{7.16}$$

where $x = \ln a$. The differential equation for the fractional density of dark energy is obtained by taking the derivative with respect to x in both sides of Eq. (7.16), yielding

$$\Omega_{\phi}^{'} = -(1+z)^{-1}\Omega_{\phi}(1-\Omega_{\phi})\left(1+\frac{2}{c}\sqrt{\Omega_{\phi}}\right), \tag{7.17}$$

where the prime denotes the derivative with respect to the redshift z. This equation has an exact solution [24] and describes the evolution of the HDE as a function of the redshift. Since $\Omega_{\phi}^{'}$ is always positive, the fraction of dark energy increases with time. From the energy conservation equation of dark energy, the equation of state of dark energy can be given [24]

$$\omega_{\phi} = -1 - \frac{1}{3}\frac{d\ln\rho_{\phi}}{d\ln a} = -\frac{1}{3}\left(1+\frac{2}{c}\sqrt{\Omega_{\phi}}\right). \tag{7.18}$$

Note that the formula $\rho_{\phi} = \frac{\Omega_{\phi}}{1-\Omega_{\phi}}\rho_{\mathrm{m}}^{0}a^{-3}$ and the differential equation of Ω_{ϕ} are used in the second equal sign.

From Eqs.(7.4),(7.6) and (7.12), we can obtain the expression for F as a function of the redshift z

$$F(z) = -\rho_{\phi}\,\omega_{\phi} = -3M_p^2\,H^2(z)\,\Omega_{\phi}(z)\,\omega_{\phi}(z). \tag{7.19}$$

Note that, since $\omega_{\phi}(z) < 0$, the above expression indicates that F is positive in this approach. If we demand that the energy density be positive, Eq.(7.4) implies that $F_X < F/2X$. Therefore, for kinetic k-essence, $F > 0$ and $F_X < 0$ imply that $w > -1$ (cf. [109]) noticing the difference in the sign convention for the energy density and the pressure). Now we focus on the reconstruction of $F(X)$ in the redshift range between $z = 0$ and $z = 1.8$ which is the current range for the supernova data. We shall do so in the light of the HDE with $c \geq 1$ as the future event horizon is only well defined when $w \geq -1$ (see [24]). As an example, we plot in Fig.7.1 some evolutions of the equation of state of the HDE. We show in the plot the cases $c = 1, 1.1, 1.2$ and 1.3. It is clear that for these cases $c \geq 1$, they always evolve in the region of $w \geq -1$.

In order to carry out the numerical evaluation which allows to find F as a function of X, we use the dimensionless variable $\mathcal{F} = F/(M_p^2 H_0^2)$. Rewriting Eq.(7.11) yields

$$X\left(\frac{d\mathcal{F}}{dz}\frac{dz}{dX}\right)^2 = \frac{k}{(M_p^2 H_0^2)^2}\,(1+z)^6, \tag{7.20}$$

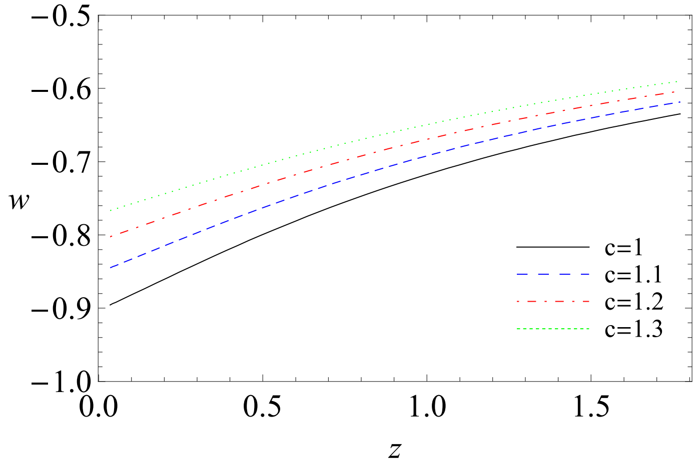

Figure 7.1: The evolutions of the equation of state of holographic dark energy. Here we take $\Omega_{\rm m0} = 0.27$, and show the cases for $c = 1$, 1.1, 1.2 and 1.3.

where the usual relation $1 + z = 1/a$ has been used. Defining the function $g(z)$ by

$$g(z) \equiv \left(\frac{d\mathcal{F}}{dz}\right)^2, \tag{7.21}$$

we can obtain the following expression which allows us to determine X as a function of z

$$\int\limits_{X_0}^{X} \left(\frac{1}{M_p^2 H_0^2}\right) \sqrt{\frac{k}{X'}} dX' = \int\limits_0^z \frac{\sqrt{g(z)}}{(1+z)^3} dz. \tag{7.22}$$

We assume that $k/X > 0$ in order to have real solutions for X. Integrating the above equation yields

$$\frac{X}{X_0}(z) = \left(\frac{1}{2}\left(\frac{M_p^2 H_0^2}{\sqrt{kX_0}}\right) \int\limits_0^z \frac{\sqrt{g(z)}}{(1+z)^3} dz + 1\right)^2, \tag{7.23}$$

which admits the following analytical solution

$$\frac{X}{X_0} = \left(\frac{\Omega_{m0}\Omega_\phi \left(c - \sqrt{\Omega_\phi}\right)}{c\left(1 - \Omega_\phi\right)\left(\frac{3}{2}\left(\Omega_{m0} - 1\right) + \Omega_{\phi 0}\left(\frac{1}{2} + \frac{\sqrt{\Omega_{\phi 0}}}{c}\right)\right)}\right)^2 \tag{7.24}$$

where X_0 and Ω_{m0} are the current values for X and Ω_m.

From Eqs. (7.19) and (7.24) we can obtain the function $\mathcal{F} = \mathcal{F}(X/X_0)$.

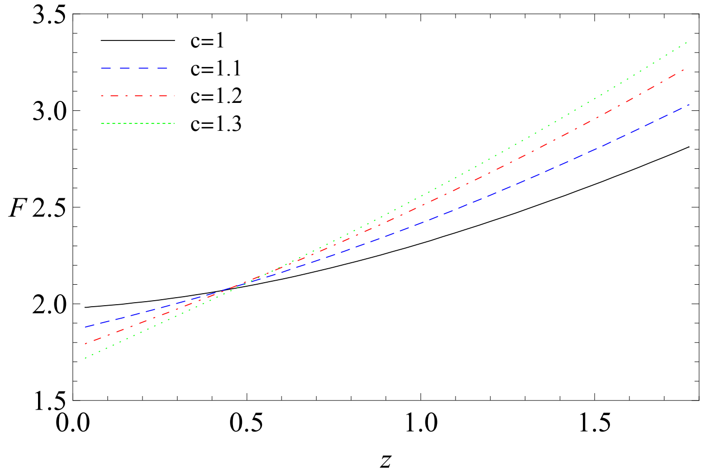

Figure 7.2: Variation of $F(z)$ in units of $M_P^2 H_0^2$. Here we take $\Omega_{\rm m0} = 0.27$, and show the cases for $c = 1$, 1.1, 1.2 and 1.3.

As me mention before, from Eq. (7.19), F must be necessarily positive and a monotonically increasing function with z within the relevant redshift range, for an accelerating universe with HDE. This behaviour is shown in Fig.7.2.

Likewise, the behaviour of X/X_0 as a function of the redshift z is showed in Fig.7.3.

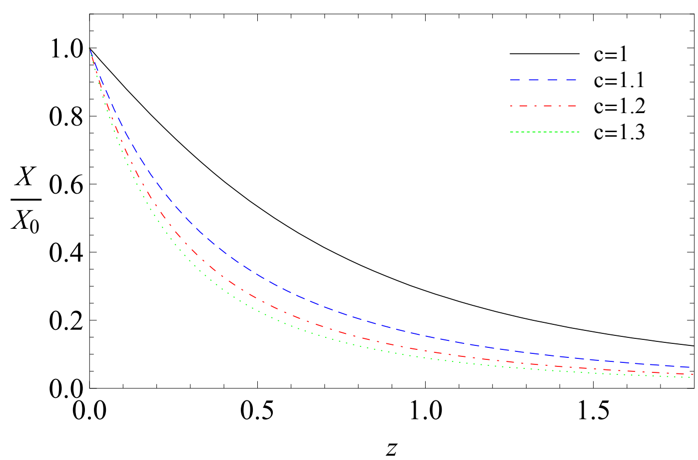

Figure 7.3: Variation of $\frac{X}{X_0}(z)$. Here we take $\Omega_{\rm m0} = 0.27$, and show the cases for $c = 1$, 1.1, 1.2 and 1.3.

The holographic kinetic k-essence, represented by the function $\mathcal{F}$ is plotted in Fig.7.4 as a function of X/X_0. From Figs. 7.3 and 7.4 we can see the dynamics of the k-essence field explicitly. F is a monotonically decreasing function of X in the relevant redshift range. This is because for $X > 0$, the sign of $\frac{F_X}{F}$ is related to the value of w_ϕ. We should emphasise that the reconstruction of $F(X)$ only involves the portion of it over which the field evolves to give the required $H(z)$. Incidentally, Figs. 7.2, 7.3 and 7.4 are very similar to the ones shown in [110] for the transient case although the author was dealing there with a non-holographic model in which the ansatz for the Hubble parameter $H(z)$ was obtained by modelling the dark energy as a generalised Chaplygin gas. We see that the reconstructed $\mathcal{F} = \mathcal{F}(X/X_0)$ is a well-behaved, single valued function.

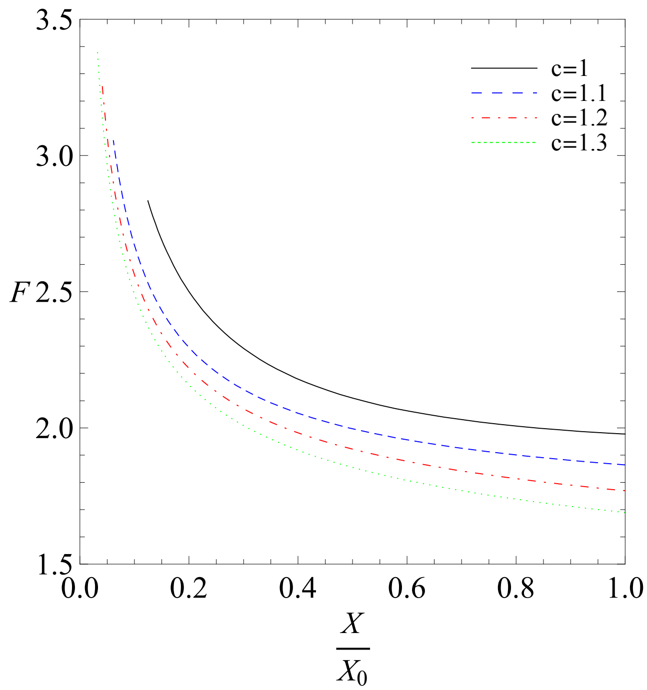

Figure 7.4: Reconstructed $F(X/X_0)$ in units of $M_P^2 H_0^2$. Here we take $\Omega_{m0} = 0.27$, and show the cases for $c = 1$, 1.1, 1.2 and 1.3.

The HDE models depend mainly on the parameter c. From Eq.(7.18), we see that the equation of state satisfies $-(1 + 2/c)/3 \leqslant w \leqslant -1/3$ due to $0 \leqslant \Omega_\phi \leqslant 1$, showing that the parameter c plays a key role in the holographic evolution of the universe.

When $c \geqslant 1$, the case we are studying, the equation of state will evolve in the region of $-1 \leqslant w \leqslant -1/3$. The value of c should be determined by cosmological observations in the holographic scenario. The case $c \geqslant 1$ is worth investigating as current observational data cannot determine the value of c accurately. In recent fit studies, different groups gave different values for c. An analysis of some recent observational data, including the gold sample of 182 SNIa, the CMB shift parameter given by the 3-year WMAP observations, and the BAO measurement from the SDSS, showed that the possibilities of $c > 1$ and $c < 1$ both exist and their likelihoods are almost equal within 3 sigma error range [111].

K-essence models with different $F(X)$ have been discussed in the literature. For the holographic kinetic k-essence model constructed in this chapter, the reconstructed $F(X)$ can be determined from Eqs.(7.19) and (7.24). If we take $c = 1$, the behaviour is similar to the cosmological constant.

If $c > 1$, the equation of state of dark energy will be always larger than -1 and therefore the universe does not enter the de Sitter phase and avoids the occurrence of a Big Rip. Thus, we see explicitly that the value of c is paramount for the HDE model as it determines the feature of the HDE as well as the ultimate fate of the universe.

7.4 Dilatonic dark energy

We consider as a starting point the four-dimensional effective low-energy string action which is generally given by

$$\mathcal{S} = \int d^4x\sqrt{-\tilde{g}}\{B_g(\phi)\tilde{R} + B_\phi^{(0)}(\phi)(\tilde{\nabla}\phi)^2 - \alpha'[c_1^{(1)}B_\phi^{(1)}(\phi)(\tilde{\nabla}\phi)^4 + ...] + O(\alpha'^2)\} \quad (7.25)$$

where ϕ is the dilaton field that controls the strength of the string coupling g_s^2 via the relation $g_s^2 = e^\phi$. Here we set $\kappa^2 = 8\pi G = 1$. The low-energy effective string theory generates higher-order derivative terms coming from α' and loop corrections (here α' is related to the string length scale λ_s via the relation $\alpha' = \lambda_s/2\pi$).

In the weak coupling regime ($e^\phi \ll 1$) the coupling functions have the dependence $B_g \simeq B_\phi^{(0)} \simeq B_\phi^{(1)} \simeq e^{-\phi}$.

We shall work in the context of the so-called runaway dilaton scenario [112] in which the coupling functions in Eq. (7.25) are given by

$$B_g(\phi) = C_g + D_g e^{-\phi} + \mathcal{O}(e^{-2\phi})\,, \quad (7.26)$$

$$B_\phi^{(0)}(\phi) = C_\phi^{(0)} + D_\phi^{(0)} e^{-\phi} + \mathcal{O}(e^{-2\phi})\,. \quad (7.27)$$

In this case $B_g(\phi)$ and $B_\phi^{(0)}(\phi)$ approach constant values as $\phi \to \infty$. Hence the dilaton gradually decouples from gravity as the field evolves towards the region $\phi \gg 1$ from the weakly coupled regime and we assume that the dilaton is effectively decoupled from gravity in the limit $\phi \to \infty$ and therefore behaves as a scalar field.

Once we assume that the dilaton behaves as a scalar field, we consider the following general 4-dimensional action

$$S = S_{grav} + S_\phi = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}R + p_D(X,\phi)\right], \quad (7.28)$$

where R is the Ricci scalar and the effective Lagrangian density $p_D(X,\phi)$ can be expressed as

$$p_D(X,\phi) = -X + de^{\lambda\phi}X^2 \tag{7.29}$$

being d a positive constant and $X = \frac{1}{2}\dot{\phi}^2$ the kinetic term of the dilaton scalar field ϕ. This is a higher-order kinetic correction to the usual kinetic term motivated by dilatonic higher-order corrections to the three-level action in low-energy effective string theory [93]. Since the $e^{\lambda\phi}$ term in Eq. (7.29) can be large for $\phi \to \infty$, the second term in Eq. (7.29) can stabilise the vacuum even if X is much smaller than the Planck scale.

In string theory we have other non-perturbative and loop corrections such as the Gauss-Bonnet (GB) curvature invariant. Further, a dark energy model based on a string-inspired Lagrangian must in general contain higher derivative terms. It is also important to acknowledge the role that the GB coupling with the scalar field may play in the late-time universe [113, 114]. Moreover, the cosmological implications of the HDE density in the Gauss-Bonnet framework have been investigated in [115]. However, in this chapter, we shall carry out the analysis for a simplified Lagrangian in order to understand the basic picture of the system. This seems to be justifiable [112], and the dilatonic dark energy model obtained [93] possesses the characteristics of a viable model of dark energy.

We assume a spatially flat FRW background spacetime $ds^2 = dt^2 - a^2(t)\, d\vec{x}^2$ (where $a(t)$ is the scale factor). Unless otherwise stated, we consider ϕ to be smooth on scales of interest so that $X = \frac{1}{2}\dot{\phi}^2 \geq 0$. The energy-momentum tensor of the dilaton is obtained from Eq. (7.28), yielding

$$T^{(\phi)}_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta S_\phi}{\delta g^{\mu\nu}} = g_{\mu\nu}p_D + p_{,X}\partial_\mu\phi\partial_\nu\phi, \tag{7.30}$$

where $p_{,X} \equiv \partial p/\partial X$. Since the energy-momentum tensor (7.30) of the dilaton scalar field is that of a perfect fluid, $T_{\mu\nu} = (\rho + p)u_\mu u_\nu + g_{\mu\nu}p$, with velocity $u_\mu = \partial_\mu\phi/\sqrt{2X}$, we have the dilaton energy density ρ_D

$$\rho_D = 2Xp_{D,X} - p_D = -X + 3de^{\lambda\phi}X^2 \tag{7.31}$$

and the Lagrangian density pressure in Eq.(7.29) corresponds to the dilaton pressure p_D. Throughout this chapter, we shall assume that the energy density is positive so that $-X + 3de^{\lambda\phi}X^2 > 0$.

We now proceed to derive the stability conditions of the dilatonic dark energy by considering small fluctuations $\delta\phi(t,\mathbf{x})$ around a background value $\phi_0(t)$ which is the solution in the FRW spacetime. Then the field $\phi(t,\mathbf{x})$ can be decomposed in the conventional form

$$\phi(t, \mathbf{x}) = \phi_0(t) + \delta\phi(t, \mathbf{x})\,. \tag{7.32}$$

Since we are interested in ultraviolet (UV) instabilities, it is not restrictive to consider a Minkowski background. Expanding $p_D(X,\phi)$ at the second order in $\delta\phi$, it is straightforward to find the Lagrangian and then the Hamiltonian for the fluctuations. The perturbed Hamiltonian reads

$$\mathcal{H} = (p_{D,X} + 2Xp_{D,XX})\frac{(\delta\dot{\phi})^2}{2} + p_{D,X}\frac{(\nabla\delta\phi)^2}{2} - p_{D,\phi\phi}\frac{(\delta\phi)^2}{2}. \tag{7.33}$$

The positive definiteness of the Hamiltonian is guaranteed if the following conditions hold

$$\xi_1 \equiv p_{D,X} + 2Xp_{D,XX} \geq 0, \quad \xi_2 \equiv p_{D,X} \geq 0, \tag{7.34}$$

$$\xi_3 \equiv -p_{D,\phi\phi} \geq 0\,. \tag{7.35}$$

When discussing the stability of classical perturbations, a quantity often used is the speed of sound c_s defined by [101]

$$c_s^2 \equiv \frac{p_{D,X}}{\rho_{D,X}} = \frac{\xi_2}{\xi_1}\,. \tag{7.36}$$

In cosmological perturbation theory c_s^2 appears as a coefficient of the k^2/a^2 term, where k is the comoving wavenumber. While the classical fluctuations may be regarded as stable when $c_s^2 > 0$, the stability of quantum fluctuations requires both the conditions $\xi_1 > 0$ and $\xi_2 \geq 0$. These two conditions prevent an instability related to the presence of negative energy ghost states. If these conditions are violated, the vacuum is unstable under a catastrophic production of ghosts and photon pairs [33, 34]. The production rate from the vacuum is proportional to the phase space integral on all possible final states. Since only a UV cut-off can prevent the creation of modes of arbitrarily high energies, this is essentially a UV instability. In our model the $e^{\lambda\phi}$ appearing in the second term of the RHS in Eq.(7.29) can be large for $\phi \to \infty$, so that such a term in Eq.(7.29) can stabilise the vacuum even if X was much smaller than the Planck scale. In particular, since in our model $\xi_1 = -1+6de^{\lambda\phi}X$ and $\xi_2 = -1 + 2de^{\lambda\phi}X$, the quantum stability is ensured for $de^{\lambda\phi}X \geq 1/2$. The equation of state for the dilaton can be written as $p_D = w_D\rho_D$ which, when rearranged, gives the equation of state parameter

$$w_D = \frac{p_D}{\rho_D} = \frac{dXe^{\lambda\phi} - 1}{3dXe^{\lambda\phi} - 1}. \tag{7.37}$$

Hence we have $w_D \geq -1$ under the condition $de^{\lambda\phi}X \geq 1/2$, which means that the phantom equation of state ($w_D < -1$) cannot be realised if we want the model to be quantum mechanically stable.

Let us study now the cosmological dynamics of the dilatonic dark energy model in the flat FRW background. As a matter fluid, with energy density ρ_m, we take both baryons and cold dark matter. The Einstein equations in this case are

$$3H^2 = \rho_D + \rho_m\,, \tag{7.38}$$
$$2\dot{H} = -(2Xp_{D,X} + \rho_m)\,, \tag{7.39}$$
$$\dot{\rho}_D + 3H(\rho_D + p_D) = 0\,, \tag{7.40}$$

where hereafter in this chapter we set $M_P = 1$. Inserting Eqs. (7.29) and (7.31) in the above equations yields

$$3H^2 = -\frac{1}{2}\dot{\phi}^2 + \frac{3}{4}de^{\lambda\phi}\dot{\phi}^4 + \rho_m\,, \tag{7.41}$$
$$2\dot{H} = \dot{\phi}^2 - de^{\lambda\phi}\dot{\phi}^4 - \rho_m\,, \tag{7.42}$$
$$\ddot{\phi}(3de^{\lambda\phi}\dot{\phi}^2 - 1) + 3H\dot{\phi}(de^{\lambda\phi}\dot{\phi}^2 - 1) + \frac{3}{4}d\lambda e^{\lambda\phi}\dot{\phi}^4 = 0\,. \tag{7.43}$$

In order to study cosmological dynamics in the presence of the dilaton scalar field and a background fluid, it is convenient to introduce the following dimensionless variables

$$x_1 \equiv \frac{\dot{\phi}}{\sqrt{6}H}\,, \quad x_2 \equiv \frac{e^{-\lambda\phi/2}}{\sqrt{3}H}\,. \tag{7.44}$$

which can be written in an autonomous form

$$\frac{\mathrm{d}x_1}{\mathrm{d}N} = \frac{3}{2}x_1\left[1 + x_1^2(dY - 1)\right] + \frac{1}{1 - 6dY}\left[3(2dY - 1)x_1 + \frac{3\sqrt{6}}{2}\lambda d x_1^2 Y\right], \tag{7.45}$$
$$\frac{\mathrm{d}x_2}{\mathrm{d}N} = -\frac{\sqrt{6}}{2}\lambda x_1 x_2 + \frac{3}{2}x_2[1 + x_1^2(dY - 1)]\,, \tag{7.46}$$

where $N = \ln a$ is the number of e-foldings which is convenient to use for the dynamics of dark energy and

$$Y \equiv \frac{x_1^2}{x_2^2} = Xe^{\lambda\phi}\,. \tag{7.47}$$

The equation of state and the fraction of the energy density for the dilaton field can now be written as

$$w_D = \frac{1 - dY}{1 - 3dY}\,, \tag{7.48}$$

$$\Omega_D = \frac{\rho_D}{3H^2} = -x_1^2 + 3d\frac{x_1^4}{x_2^2}\,. \tag{7.49}$$

The condition for the stability of quantum fluctuations corresponds to $dY \geq 1/2$. The following fixed points are relevant for viable cosmological evolution:

(a) Matter point: $(x_1, x_2) = (0, 1/2)$. This satisfies $w_D = -1$, $\Omega_D = 0$ and $\Omega_m = 1$.

(b) Accelerated point: $(x_1, x_2) = (-\sqrt{6}\lambda f_-(\lambda)/4, 1/2 + \lambda^2 f_+(\lambda)/16)$, where

$$f_\pm \equiv 1 \pm \sqrt{1 + 16/(3\lambda^2)}\,. \tag{7.50}$$

This satisfies $w_D = (-8 + \lambda^2 f_+(\lambda))/(8 + 3\lambda^2 f_+(\lambda))$, $\Omega_D = 1$ and $\Omega_m = 0$. The cosmic acceleration occurs for $-1 \leq w_D < -1/3$, i.e., $1/2 \leq dY < 2/3$. This corresponds to the condition $0 \leq \lambda^2 f_+(\lambda) < 8/3$, i.e.,

$$0 \leq \lambda < \sqrt{6}/3. \tag{7.51}$$

It can be shown that this accelerated point is stable for $0 \leq \lambda < \sqrt{3}$ [93]. Hence the stability of the accelerated point is ensured under the condition (7.51).

We also have other fixed points. For example, there is another accelerated point $(x_1, x_2) = (-\sqrt{6}\lambda f_+(\lambda)/4, 1/2 + \lambda^2 f_-(\lambda)/16)$, but this corresponds to the quantum instability region $dY < 1/2$ (i.e. the phantom equation of state $w_D < -1$). During the matter era we also have the scaling solution with $(x_1, x_2) = (\sqrt{6}/(2\lambda), 1)$, $\Omega_D = 3/\lambda^2$, and $w_D = 0$. However, the existence of a viable scaling matter era requires the condition $\lambda > \sqrt{3}$, which is not compatible with the condition (7.51).

We shall study the stability of the fixed points in the case $d = 1$. The eigenvalues of the matrix $\mathcal{M}$ were numerically evaluated in Ref. [116] and it was shown that the determinant of the matrix $\mathcal{M}$ for the point $(x_1, x_2) = (-\sqrt{6}\lambda f_+(\lambda)/4, 1/2 + \lambda^2 f_-(\lambda)/16)$ is negative with negative real parts of μ_1 and μ_2. Hence this phantom fixed point is a stable spiral. As already mentioned, the point (b) is a stable node for $0 < \lambda < \sqrt{3}$, whereas it is a saddle point for $\lambda > \sqrt{3}$. This critical value $\lambda_* = \sqrt{3}$ is computed by setting the determinant of $\mathcal{M}$ to be zero. The point $(x_1, x_2) = (\sqrt{6}/(2\lambda), 1)$ is physically meaningful for $\lambda > \sqrt{3}$ because of the condition $\Omega_\phi < 1$, and it is a stable node [116]. Hence the point $(x_1, x_2) = (\sqrt{6}/(2\lambda), 1)$ is stable when the point (b) is unstable and vice versa. It was shown in Ref. [117] that this property holds for all scalar-field models which possess scaling solutions. We recall that the point $(x_1, x_2) = (-\sqrt{6}\lambda f_+(\lambda)/4, 1/2 + \lambda^2 f_-(\lambda)/16)$ is not stable at the quantum level. The above discussion shows that the only viable attractor which satisfies the conditions of an accelerated expansion and the quantum stability is the point (b). Finally, we recall that the sound speed of the dilatonic model is smaller that the speed of light because the condition $p_{,XX} \geq 0$ holds. The sound speed squared in this case is given by

$$c_s^2 = \frac{2dY - 1}{6dY - 1}. \tag{7.52}$$

The condition (7.51) for the existence of the late-time accelerated point gives $1/2 \leq dY < 2/3$. Hence the sound speed runs in the interval

$$0 \leq c_s < 1/3 \tag{7.53}$$

which means that this model does not violate causality.

7.5 Holographic dilatonic dark energy model

We shall proceed with our study in the light of the HDE with $c \geq 1$ as the future event horizon is only well defined when $w_D \geq -1$ (see [24]) and we also want to ensure quantum stability.

In order to build our holographic model, we impose the holographic nature to the dilatonic dark energy, i.e., we identify ρ_D with ρ_Λ, this is the same procedure we followed in the case of the kinetic k-essence.

We consider a universe filled with a matter component ρ_m and a holographic dilatonic component ρ_D, the Friedmann equation (18) can be equivalently expressed as

$$H(z) = H_0 \left(\frac{\Omega_{m,0}(1+z)^3}{1 - \Omega_D} \right)^{1/2} \tag{7.54}$$

where $z = (1/a) - 1$ is the redshift of the universe. From the definition of the HDE and the definition of the future event horizon, we find

$$\int_a^\infty \frac{da'}{Ha'^2} = \int_x^\infty \frac{dx}{Ha} = \frac{c}{\sqrt{\Omega_D} Ha} \tag{7.55}$$

The Friedmann equation (7.54) implies

$$\frac{1}{Ha} = \sqrt{a(1-\Omega_D)}\frac{1}{H_0\sqrt{\Omega_{m,0}}} \tag{7.56}$$

Inserting (7.56) into (7.55), we arrive at

$$\int_x^\infty e^{x'/2}\sqrt{1-\Omega_D}dx' = ce^{x/2}\sqrt{\frac{1}{\Omega_D}-1}, \tag{7.57}$$

where $x = \ln a$. The differential equation for the fractional density of dark energy is obtained by taking the derivative with respect to x in both sides of equation (7.57), yielding

$$\Omega_D^{'} = -(1+z)^{-1}\Omega_D(1-\Omega_D)\left(1+\frac{2}{c}\sqrt{\Omega_D}\right), \tag{7.58}$$

where the prime denotes the derivative with respect to the redshift z. This equation has an exact solution [24] and describes the evolution of the HDE as a function of the redshift. Since $\Omega_D^{'}$ is always positive, the fraction of dark energy increases with time. From the energy conservation equation of dark energy, the equation of state parameter of dark energy can be expressed as [24]

$$\omega_D = -1 - \frac{1}{3}\frac{d\ln\rho_D}{d\ln a} = -\frac{1}{3}\left(1+\frac{2}{c}\sqrt{\Omega_D}\right). \tag{7.59}$$

Note that the formula $\rho_D = \frac{\Omega_D}{1-\Omega_D}\rho_{m,0}a^{-3}$ and the differential equation of Ω_D, Eq.(7.58), are used for the second equality.

The use of Eqs. (7.54),(7.31) and (7.37) allows the derivation of the kinetic term X in terms of holographic quantities

$$\frac{X}{\rho_{cr,0}} = \frac{\Omega_D\Omega_{m,0}(1-3w_D)(1+z)^3}{2(1-\Omega_D)}, \tag{7.60}$$

where Ω_D and w_D are given by Eqs.(7.58) and (7.59) respectively, and $\rho_{cr,0} = 3H_0^2$ is the critical density at the present epoch.

Moreover, from the definition of the kinetic term $X = \frac{1}{2}\dot{\phi}^2$ and Eq. (7.60), we can deduce the derivative of the holographic dilatonic scalar field ϕ with respect to the redshift z

$$\phi' = \mp\frac{\sqrt{3\Omega_D(1-3w_D)}}{1+z}, \tag{7.61}$$

where the sign is in fact arbitrary as it can be changed by a redefinition of the field $\phi \to -\phi$. The evolutionary form of the holographic dilatonic field can be easily obtained by integrating

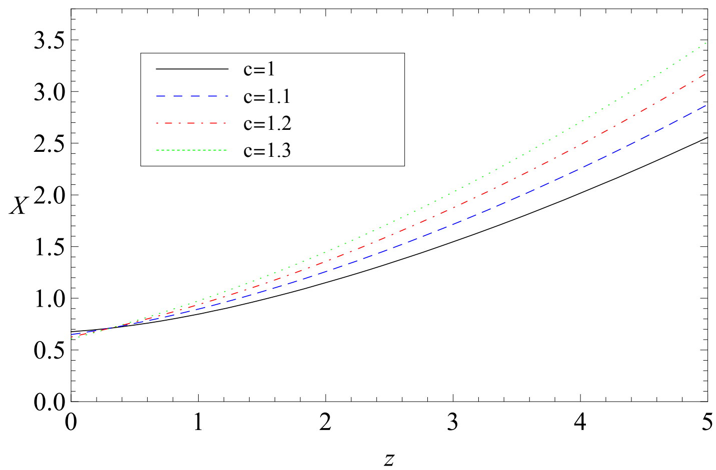

Figure 7.5: Variation of $X(z)$, where X is in units of $3H_0^2$. We take here $\Omega_{m,0} = 0.27$ and show the cases for $c = 1, 1.1, 1.2, 1.3$.

the above equation numerically from $z = 0$ to a given value z. The field amplitude at the present epoch ($z = 0$) is taken to vanish, $\phi(0) = 0$. Changing this initial value is equivalent to a displacement in ϕ by a constant value $\phi_0 = \phi(z = 0)$, which does not affect the shape of the field.

As already explained in a previous section in this chapter, the parameter c plays an essential role in describing the evolution of the HDE model and should be determined by cosmological observations. From Eq. (7.59) we see that the equation of state parameter satisfies $-(1 + 2/c)/3 \leq w_D \leq -1/3$ due to $0 \leq \Omega_D \leq 1$. If $c = 1$, the dark energy equation of state parameter would asymptote to that of a cosmological constant and the universe would enter the de Sitter phase in the future; if $c > 1$, the equation of state parameter of dark energy would always be greater than -1, behaving as quintessence dark energy; if $c < 1$, the equation of state parameter of HDE would be initially greater than -1, but it would decrease and eventually cross the phantom divide line ($w_D = -1$) as the universe expands, acting as a quintom.

The best-fit analysis on the HDE model, by using the latest observational data including the Union+CFA3 sample of 397 Type Ia supernovae (SNIa), the shift parameter of the cosmic microwave background (CMB) given by the five-year Wilkinson Microwave Anisotropy Probe (WMAP5) observations, and the baryon acoustic oscillations (BAO) measurement from the Sloan Digital Sky Survey (SDSS) favours quintom behaviour slightly. However, quintessence-like behaviour is also still allowed with the present data [32], [35]. That is why the case $c \geq 1$ is worth investigating in detail. In addition, [32] shows that $c < 1.2$ at more than 3σ, which is consistent with the possible theoretical limit of the parameter c

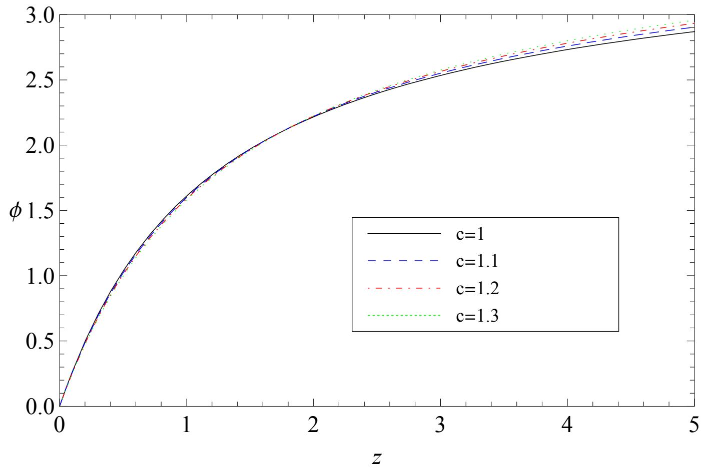

Figure 7.6: The evolution of the dilaton scalar field $\phi(z)$ with the $(-)$ sign in Eq.(7.61). We take here $\Omega_{m,0} = 0.27$ and show the cases for $c = 1, 1.1, 1.2, 1.3$.

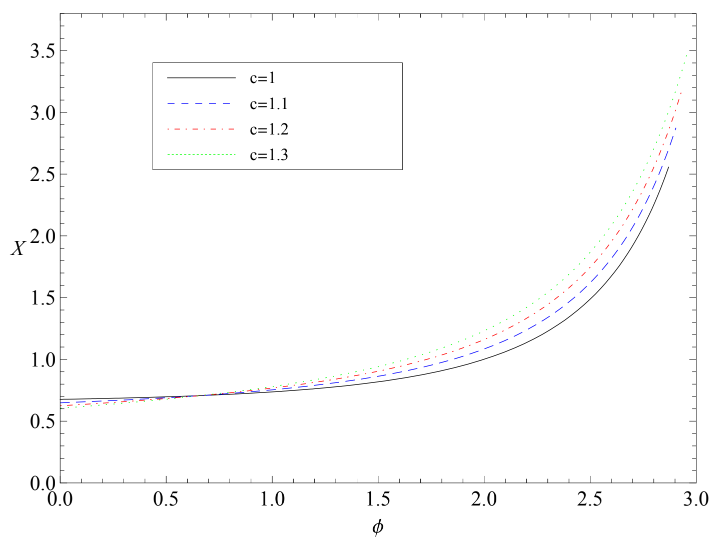

Figure 7.7: Reconstructed X for the holographic dilaton where X is in units of $3H_0^2$. We take here $\Omega_{m,0} = 0.27$ and show the cases for $c = 1, 1.1, 1.2, 1.3$.

from the weak gravity conjecture (see [118]). It was also found that the HDE model fits mildly better than the ΛCDM, but with the data available at present the difference is not significant.

The holographic evolution of the kinetic term can be obtained numerically and it is shown in Fig.7.5 where we can see that X is a positive and monotonically increasing function with z for an accelerating universe with HDE. Likewise, the behaviour of $\phi(z)$, obtained through Eq.(7.61), is displayed in Fig.7.6.

The holographic dilatonic dark energy, represented by X, is plotted in Fig.7.7 as a function of ϕ. From Figs. 7.6 and 7.7 we can see the dynamics of the field explicitly. Selected curves are plotted for the cases of $c = 1.0, 1.1, 1.2$ and 1.3, and the present fractional matter density is chosen to be $\Omega_{m,0} = 0.27$. Given that the kinetic term decreases gradually with the cosmic evolution, the equation of state parameter of the dilaton w_D tends to negative values close to -1 according to Eq. (7.59) as $\dot{\phi} \to 0$. As a result $dw_D/d\text{lna}< 0$. Note that $\phi(z)$ increases with z but becomes finite at high redshift. This means that ϕ decreases as the universe expands. Similar behaviour was obtained in [94] for the holographic quintessence and in [96] for the holographic tachyon model.

7.6 Conclusions

By assuming that the scalar field models of dark energy are effective theories of an underlying theory of dark energy, we can use scalar field models to mimic the evolving behaviour of the HDE. In this chapter, we have discussed two holographic models of dark energy with the future event horizon as infrared cut-off. This has been done by establishing a correspondence between the HDE model and the kinetic k-essence and the dilaton scalar fields, respectively. The holographic kinetic k-essence function $F(X)$ and the dilatonic kinetic term as well as their dynamics have been reconstructed for a redshift range between $z = 0$ and $z \leq 2$. We have also carried out a detailed analysis of their evolution and explore their cosmological consequences in the region $-1 < w < -1/3$, which is the allowed region for these models when $c \geq 1$ and shown that the predictions from the resulting models adjust perfectly well to the measured parameters for the observed accelerated expansion of our universe.

Chapter 8

Interacting holographic dark energy: tachyon model

8.1 Introduction

In this chapter, we extend the HDE further by developing a holographic tachyon model of dark energy with interaction between the components of the dark sector. The correspondence between the tachyon field and the holographic dark energy densities allows the reconstruction of the potential and the dynamics of the tachyon scalar field in a flat Friedmann-Robertson-Walker universe. We show that this model can describe the observed accelerated expansion of our universe with a parameter space given by the most recent observational results.

8.2 Tachyon scalar field

The fact that the tachyon can act as a source of dark energy with different potential forms have been widely discussed in the literature [119–122]. The tachyon can be described by an effective field theory corresponding to a tachyon condensate in a certain class of string theories with the following effective action [123, 124]

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{16\pi G} - V(\phi)\sqrt{1+g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi}\right], \tag{8.1}$$

where $V(\phi)$ is the tachyon potential and R the Ricci scalar. The physics of tachyon condensation is described by the above action for all values of ϕ provided the string coupling and the second derivative of ϕ are small. The corresponding energy-momentum tensor of the tachyon field has the form

$$T_{\mu\nu} = \frac{V(\phi)\partial_\mu\phi\partial_\nu\phi}{\sqrt{1+g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi}} - g_{\mu\nu}V(\phi)\sqrt{1+g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi}. \tag{8.2}$$

In the flat FRW background the energy density ρ_t and the pressure p_t are given by

$$\rho_t = -T_0{}^0 = \frac{V(\phi)}{\sqrt{1-\dot{\phi}^2}}, \tag{8.3}$$

$$p_t = T_i{}^i = -V(\phi)\sqrt{1-\dot{\phi}^2}, \tag{8.4}$$

where no summation over repeated indices is assumed and the dot stands for the derivative with respect to cosmic time.

From the Friedmann equation $3M_P^2H^2 = \rho_t$ and the continuity equation $\dot{\rho}_t + 3H(\rho_t + p_t) = 0$ we obtain the following equations of motion:

$$H^2 = \frac{V(\phi)}{3M_P^2\sqrt{1-\dot{\phi}^2}}, \tag{8.5}$$

$$\frac{\ddot{\phi}}{1-\dot{\phi}^2} + 3H\dot{\phi} + \frac{1}{V}\frac{\mathrm{d}V}{\mathrm{d}\phi} = 0\,. \tag{8.6}$$

Combining these equations gives

$$\frac{\ddot{a}}{a} = \frac{V(\phi)}{3M_P^2\sqrt{1-\dot{\phi}^2}}\left(1-\frac{3}{2}\dot{\phi}^2\right). \tag{8.7}$$

Hence an accelerated expansion occurs for $\dot{\phi}^2 < 2/3$.

From Eqs. (8.3) and (8.4) we obtain the tachyon equation of state parameter

$$w_t = \frac{p_t}{\rho_t} = \dot{\phi}^2 - 1. \tag{8.8}$$

In order to have a real energy density for the tachyon we require $0 < \dot{\phi}^2 < 1$ which implies, from Eq. (8.8), that the equation of state parameter is constrained to $-1 < w_t < 0$. Hence, irrespective of the form of the potential, the tachyonic scalar field cannot achieve an equation of state parameter that enters the phantom regime. The tachyon field can be simply classified as k-essence because it belongs to a class of the action (7.2). However, in order to achieve $w_t \approx -1$ we require that $\dot{\phi}^2 \ll 1$. This scenario is different from k-essence in the sense that the kinetic energy of the tachyon needs to be suppressed to have cosmic acceleration.

8.3 Interacting dark energy

In the previous chapter, we dealt with the HDE model and its connection with scalar fields [125,126]. A further development would be to consider a possible interaction between dark matter and the dark energy. Interacting models were first proposed by Wetterich to lower down the value of the cosmological term [88]. Later on it was proved to be efficient in alleviating the cosmic coincidence problem [127,128] unlike the ΛCDM which cannot address it. Furthermore, it has also been suggested that the interaction is not only likely but inevitable [129]. In the context of HDE, models featuring an interaction were first advanced by Horvat [130]. One advantage of the consideration of the interaction in the HDE scenario is that it can be used to avoid the future Big Rip singularity [27,131]. On the other hand, ignoring the interaction may result in a misled interpretation of the data regarding the equation of state of dark energy. In fact, it has been shown that a measured phantom equation of state may be mimicked by an interaction [132,133]. Another issue is the cosmic age problem [134]. It is known that the age of old luminous objects at high redshifts can constrain cosmological models by requiring that their age at the redshift they are observed does not exceed the age of the universe at that redshift. In particular, the age of the quasar APM 08279+5255 lies slightly further than 1σ beyond the age of the ΛCDM model at $z = 3.91$. The introduction of an interaction between dark sectors may be helpful to alleviate the cosmic age problem as shown in [135].

It is usually assumed that both dark matter and dark energy only couple gravitationally. However, given their unknown nature and that the underlying symmetry that would set the interaction to zero is still to be discovered, an entirely independent behaviour between the dark sectors would be very special indeed. Assuming that the dark energy is a field, it would be more natural for it to couple with the remaining fields of the theory, in particular with dark matter, as it is quite a general fact that different fields generally couple. Moreover, since dark energy gravitates, it must be accreted by massive compact objects such as black holes and, in a cosmological context, the energy transfer from dark energy to dark matter may be small but non-vanishing. In addition, an interaction leads to possibly important corrections of the non-interacting configuration. For instance, for a given, maybe dynamical, negative equation of state, the interaction manifest itself in third order in the redshift in the luminosity distance of type Ia supernovae [136]. However, it does not influence directly the leading orders. On the other hand, the coupling is expected to modify the isothermal Maxwell-Boltzmann velocity distribution of weakly interacting massive particles in the galaxy halos [137] whereby the average dark matter velocity can augment significantly.

It is therefore important to take the possible existence of the coupling into account when devising experiments searching for dark matter given that the detection rates strongly depend on the aforementioned velocity [138].

Although, as we write, the available empirical data cannot discriminate between the existence of a small interaction and its total absence, a couple of analysis seem to favour the former possibility: (i) As is to be expected, the interaction alters the time required for a self-gravitating, collapsing, structure to reach equilibrium as well as the equilibrium configuration itself. Therefore the Layzer-Irvine equation [139, 140] needs to be generalised to take into account the interaction. In this connection, from the study of the dynamics of 33 relaxed galaxy clusters -for which reliable x-ray, weak lensing and optical data are available-, it has been reported a small but not vanishing interaction [141, 142]. (ii) Since the interaction modifies the rate of evolution of the metric potentials, the integrated Sachs-Wolfe (ISW) component of the CMB radiation is enhanced. In fact, it has been recenly disclosed that the late ISW effect has the unique ability to provide an insight into the coupling [143]. The cross-correlation of galaxy catalogs with CMB maps also suggests a small interaction [144]. A number of studies have been devoted to analyse the constraints on the interaction from the probes of the cosmic expansion history by using the WMAP, SNIa, BAO and SDSS data, etc. [143, 145–151]. Complementary probes of the coupling have been carried out in the study of the growth of cosmic structure [152–155]. It has been also found that a non-zero interaction leaves a clear change in the growth index [152, 153].

Further, models showing interaction comply well when compared with data from the CMB [156] and matter distribution at large scales [157]. This indicates that the possibility of having an interaction between dark matter and dark energy must be taken seriously. Moreover, the interaction may give rise to fluctuations in the count of galaxy clusters with redshift [158].

If dark energy couples to dark matter through some interaction, this affects the past expansion history of the universe as well as the cosmic structure formation. The matter density, ρ_m, drops more slowly than a^{-3}. A slower matter density evolution fits the supernovae data as well as the ΛCDM concordance model does [128]. The interaction also alters the age of the universe, the evolution of matter and radiation perturbations and gives rise to a different matter and radiation power spectra. It has been found that an appropriate interaction between dark energy and dark matter can influence the perturbation dynamics and affect the lowest multipoles of the CMB angular power spectrum [159, 160]. Thus, it could be inferred from the expansion history of the universe, as manifested in the supernovae data together with CMB and large-scale structure [161]. Furthermore it was suggested that the dynamical equilibrium of collapsed structures such as clusters would be modified due to the coupling between dark energy and dark matter [142, 162]. In the absence of a fundamental theory for dark energy, the coupling term cannot be derived from microphysics. Most studies on the interaction between dark sectors rely either on the assumption of interacting fields from the outset [127, 132], or from phenomenological requirements [163]. The aforesaid interaction has also been considered from a thermodynamical perspective [164, 165] and has been shown that the second law of thermodynamics imposes an energy transfer from dark energy to dark matter. Further, the use of the Layzer-Irvine equation on nearly one hundred galaxy clusters strongly supports this view [166]. Other authors have analysed the possibility of having dark matter decaying into dark energy but it is required to have at least one of the fluids with a non null chemical potential, an assumption that which we believe it is not completely acceptable as it introduces too many unjustified components, and also relies on the assumption of a nearly standard evolution of pertubations on interacting dark energy models [167]. On the other hand, having a energy transfer from dark matter to dark energy would worsen the coincidence problem [168].

In [169], Zhang et al took advantage of the successful HDE model and used the tachyon scalar field as an effective description of an underlying theory of dark energy. In this chapter, in view of the indications that suggest an interaction between the components of the dark sector, we extend their work by incorporating a coupling between dark matter and dark energy [170]. Tachyonic fields have the attractive feature that may describe a larger variety of cosmological evolutions than quintessence fields [171]. The holographic tachyon model of dark energy was also investigated in [172] and the interacting tachyon dark energy was first studied in [173].

8.4 Interacting holographic tachyon model

In order to impose the holographic nature to the tachyon, we should identify ρ_{t} with ρ_Λ. The total energy density is $\rho = \rho_m + \rho_t$, where ρ_m and ρ_t are the matter and tachyon energy densities, respectively. Given that the matter component is mainly contributed by the cold dark matter and that, except in chameleon models, it is generally assumed that baryons do not interact with the dark sector, we shall ignore the contribution of the baryon matter here, also because of the tight constraints imposed by local gravity measurements [174, 175]. We leave radiation outside the interaction because otherwise the photons would not longer follow geodesics which would affect precise measurements of deviations of radar signals grazing the sun. Therefore, we consider a spatially flat FRW universe filled with dark matter and HDE. The Friedmann equation reads

$$3M_P^2H^2 = \rho_{\rm m} + \rho_{\rm t}. \tag{8.9}$$

In the case of an interaction between HDE and dark matter, their energy densities no longer satisfy independent conservation laws. They obey instead

$$\dot{\rho}_m + 3H\rho_m = Q\ , \tag{8.10}$$

$$\dot{\rho}_t + 3H(1+\omega_t)\rho_t = -Q, \tag{8.11}$$

where Q is an interaction term whose form is not unique.

The ratio of dark matter to dark energy, $r = \frac{\rho_m}{\rho_t}$, satisfies

$$\dot{r} = 3Hrw_t + \frac{Q}{\rho_t}(1+r). \tag{8.12}$$

Considering $w_t < -\frac{1}{3}$ and Eq. (8.12) we can see that in the non-interacting tachyon model $\dot{r} < -Hr$ and $\ddot{a} > 0$ cannot be achieved simultaneously (see [176]). Therefore, in contrast to the recent data which indicate $r \sim \mathcal{O}(1)$, $r \to 0$ eventually. This can be considered as an important hint for the need of interacting dark energy.

The expression for Q must be small (at least lower than $3H\rho_m$) because if it were large and positive, dark energy would not dominate the expansion today. On the other hand, if Q were large and negative, the Universe would have been dominated by dark energy practically from the outset and galaxies would not have formed. By inspecting the left hand side of Eqs. (8.4) and (8.11), it must be a function of the energy densities multiplied by a quantity with units of inverse of time for which we take the Hubble factor as it seems a natural choice. Therefore, we end up with an expression such as $Q = Q(H\rho_m, H\rho_t)$. If we expand this function as a power law and keep just the first term, we have $Q \simeq \lambda_m H\rho_m + \lambda_t H\rho_t$. Given the absence of information about the coupling, it makes sense to work with just one parameter, so the three possible choices are: $\lambda_m = 0$, $\lambda_t = 0$ and $\lambda_m = \lambda_t$. Here in this chapter we consider the latter choice form

$$Q = 3b^2H(\rho_{\rm m} + \rho_{\rm t}), \tag{8.13}$$

where b^2 is the coupling constant and $3H$ is attached for dimensional consistency.

This particular interaction term was first introduced on phenomenological grounds in the study of a suitable coupling between a quintessence scalar field and a pressureless cold dark matter component in order to alleviate the coincidence problem [6]. In addition, the cosmic age problem associated with the old high-z quasar APM 08279 + 5255 and the oldest globular cluster M 107, both being difficult to accommodate in the ΛCDM model, are substantially alleviated with this choice for Q as the resulting interacting model can predict a cosmic age consistent with observations which is much greater than that of ΛCDM model at any redshift [135].

The term b^2 gauges the intensity of the coupling, being $b^2 = 0$ the absence of interaction. Apart from this, it measures to what extent the different evolution of the dark matter due to its interaction with the dark energy gives rise to a different expansion history of the universe. A positive b^2 corresponds to a decay of dark energy into dark matter. In fact, it can be seen that the coincidence problem is alleviated in the IHDE model, unlike the ΛCDM one which does not have this advantage [159]. Furthermore, its observational signatures were recently investigated and this model was found to be mildly favoured over the ΛCDM one [32].

Combining the definition of HDE Eq. (2.8) and that of the future event horizon (2.9) we take the derivative with respect to $x = \ln a$ and obtain

$$\rho_t' \equiv \frac{d\rho_t}{dx} = -6M_p^2 H^2 \Omega_t (1 - \frac{\sqrt{\Omega_t}}{c}), \tag{8.14}$$

where $\Omega_t = \rho_t/(3M_p^2 H^2)$. Given that, from the definition of the Hubble parameter, $\dot{\rho}_t \equiv d\rho_t/dt = \rho_t' H$ and making use of the Friedmann equation (8.9), Eq. (8.11) can be written as

$$\rho_t' + 3(1 + w_t)\rho_t = -9M_p^2 b^2 H^2. \tag{8.15}$$

Combining the last two equations, we are led to the equation of state parameter of this IHDE model,

$$w_t = -\frac{1}{3} - \frac{2}{3}\frac{\sqrt{\Omega_t}}{c} - \frac{b^2}{\Omega_t}. \tag{8.16}$$

This is the equation we shall use throughout this chapter. However, other authors [177] argued that

$$w_t^{\text{eff}} = w_t + \frac{b^2}{\Omega_t} = -\frac{1}{3} - \frac{2}{3}\frac{\sqrt{\Omega_t}}{c} \tag{8.17}$$

should be used instead but this issue is not settled yet [178].

We must mention, however, that when the interaction between dark components is present, the situation may become somewhat ambiguous because the equation of state parameter w_t loses its ability to classify dark energies definitely, owing to the fact that now dark energy and dark matter are entangled. Under these conditions, concepts such as quintessence or phantom are not as clear as usual. Even though, we can still use these

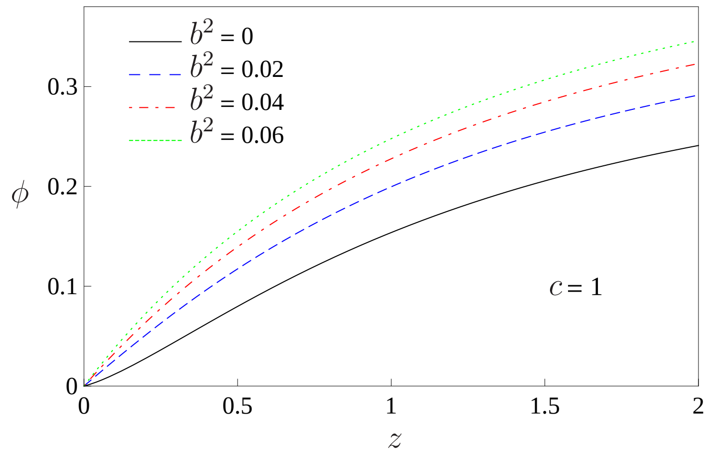

Figure 8.1: The evolution of $\phi(z)$, where ϕ is in units of H_0^{-1}, for a fixed c and different values of the coupling with $\Omega_{m0} = 0.27$.

conceptions in an undemanding sense as the interacting term is very weak according to observations.

Inserting Eq. (8.16) into Eq. (8.15) and using the definition of Ω_t, we arrive at

$$\frac{H'}{H} = -\frac{\Omega_t'}{2\Omega_t} + \frac{\sqrt{\Omega_t}}{c} - 1. \tag{8.18}$$

On the other hand, replacing $\dot{H} = H'H$ and $p_t = w_t\rho_t$ into the derivative of the Friedmann equation with respect to cosmic time $\dot{H} = -\frac{1}{2M_p^2}(\rho + p)$ (where ρ and p are the total energy density and pressure respectively), we have

$$\frac{H'}{H} = \frac{1}{2}\Omega_t + \frac{\Omega_t^{3/2}}{c} + \frac{3}{2}b^2 - \frac{3}{2}. \tag{8.19}$$

If we combine now last two equations, we find the evolution equation for Ω_t

$$\frac{d\Omega_t}{dx} = \Omega_t(1 - \Omega_t)\left(1 + \frac{2\sqrt{\Omega_t}}{c} - \frac{3b^2}{1 - \Omega_t}\right), \tag{8.20}$$

which governs the whole dynamics of the IHDE model.

Since $\frac{d}{dt} = H\frac{d}{dx} = -H(1+z)\frac{d}{dz}$ we can rewrite the above equation with respect to z as

$$\frac{d\Omega_t}{dz} = -(1+z)^{-1}\Omega_t(1 - \Omega_t)\left(1 + \frac{2\sqrt{\Omega_t}}{c} - \frac{3b^2}{1 - \Omega_t}\right). \tag{8.21}$$

Therefore, the differential equation for the Hubble parameter $H(z)$ can be expressed as

$$\frac{dH}{dz} = -(1+z)^{-1}H\left(\frac{1}{2}\Omega_t + \frac{\Omega_t^{3/2}}{c} + \frac{3}{2}b^2 - \frac{3}{2}\right). \tag{8.22}$$

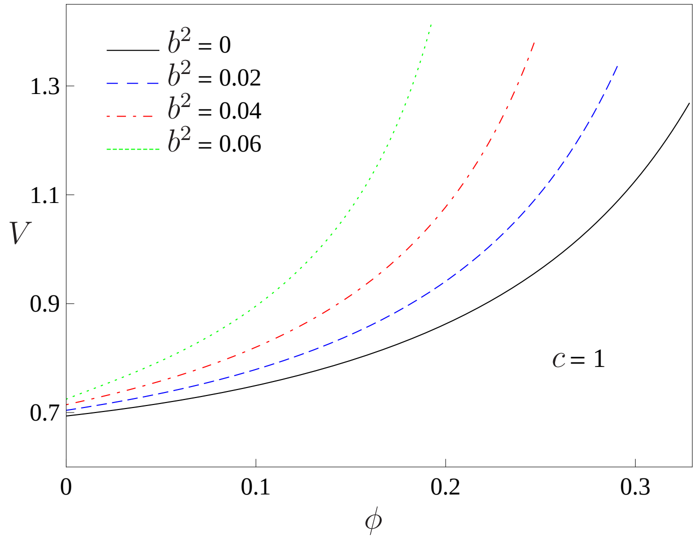

Figure 8.2: The potential for the interacting holographic tachyon model, where ϕ is in units of H_0^{-1} and $V(\phi)$ in $\varrho_{cr,0}$, for a fixed c and different values of the coupling. Here we have chosen $\Omega_{m0} = 0.27$.

The above equations can be solved numerically to obtain the evolution of Ω_t and H as a function of the redshift.

Using Eqs. (8.3), (8.8) and (8.22), we derive the interacting holographic tachyon potential

$$\frac{V(\phi)}{\rho_{cr,0}} = H^2 \Omega_t \sqrt{-w_t}, \tag{8.23}$$

where Ω_t and w_t are respectively given by Eqs. (8.21) and (8.16), being $\rho_{cr,0} = 3M_p^2 H_0^2$ the critical energy density of the universe at the present epoch. Besides, using Eqs. (8.8) and (8.22), the derivative of the interacting holographic tachyon scalar field ϕ with respect to the redshift z can be expressed as

$$\frac{\phi'}{H_0^{-1}} = \pm \frac{\sqrt{1+w_t}}{H(1+z)}. \tag{8.24}$$

The sign is in fact arbitrary as it can be changed by a redefinition of the field $\phi \to -\phi$.

The above equation cannot be solved analytically, however, the evolutionary form of the interacting holographic tachyon field can be easily obtained integrating it numerically from $z = 0$ to a given value z.

The field amplitude at the present epoch ($z = 0$) is taken to vanish, $\phi(0) = 0$. Changing

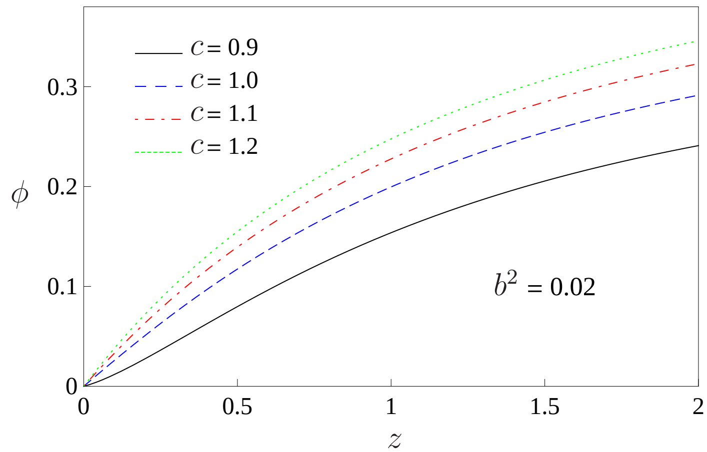

Figure 8.3: The evolution of $\phi(z)$, where ϕ is in units of H_0^{-1}, for a fixed coupling and different values of c. As is usual, here we have considered $\Omega_{m,0} = 0.27$.

this initial value is equivalent to a displacement in ϕ by a constant value $\phi_0 = \phi(z=0)$, which does not affect the shape of the field. We note that Eqs. (8.23) and (8.24) are formally the same as in [169], but $H(z)$ is different in our case due to the interaction which modifies the expansion history of the universe and the density perturbation evolution, changing the growth history of cosmological structures. As already discussed in [179] the interaction Q is very weak and positive and the parameters b^2 and c are not totally free; they need to satisfy some constraints. Following the latest observational results for the IHDE models [32, 161, 178], we take $0 \le b^2 \le 0.06$ and $\sqrt{\Omega_t} < c < 1.255$, where the lower bound of c comes from the second law of thermodynamics. The interaction coupling has an upper limit because of the evolutionary behaviour of the HDE [160]. As it can be seen in Fig. 8.5, where the dependence of the deceleration parameter

$$q = -\frac{\ddot{a}}{aH^2} = \frac{1}{2} + \frac{3}{2}w_\phi\Omega_\phi = \frac{1}{2}\left(1 - 3b^2 - \Omega_\phi - \frac{2}{c}\Omega_\phi^{3/2}\right) \tag{8.25}$$

on the coupling for a fixed c is shown, the interaction has an appreciable effect on the acceleration history of the universe. For a fixed parameter c, the cosmic acceleration starts earlier for the cases with interaction than the one without coupling as dark energy dominates earlier. This result was also previously obtained by other authors [132, 159, 160]. Moreover, for larger coupling between dark energy and dark matter, the acceleration starts earlier. However, the cases with smaller coupling will get larger acceleration finally in the far future. Besides, the cases with a fixed small b^2 and various values of c are also interesting. The universe starts to accelerate earlier when c is larger for the same coupling b^2, but finally a smaller c will lead to a larger acceleration [180].

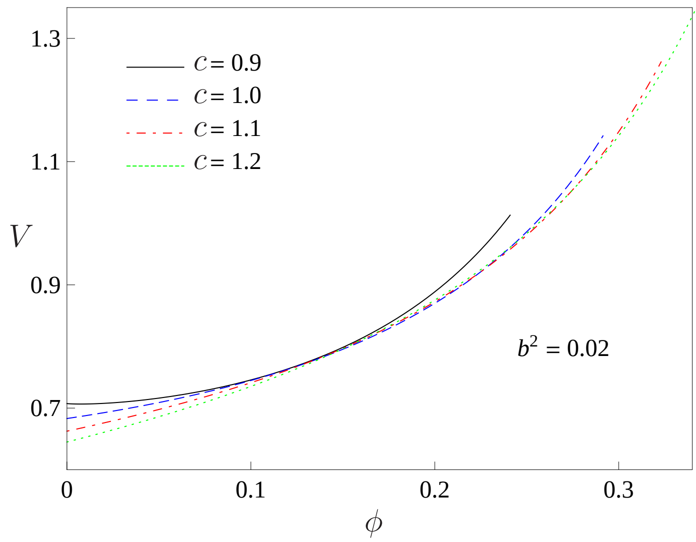

Figure 8.4: The potential for the interacting holographic tachyon model, where ϕ is in units of H_0^{-1} and $V(\phi)$ in $\varrho_{cr,0}$, for a fixed coupling and different values of c with $\Omega_{m,0} = 0.27$.

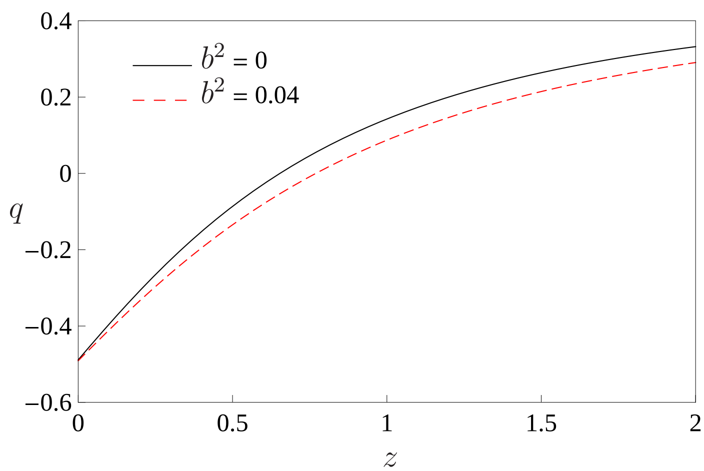

Figure 8.5: Evolution of the deceleration parameter q with and without interaction for a fixed parameter $c = 1$. We take here $\Omega_{t,0} = 0.73$.

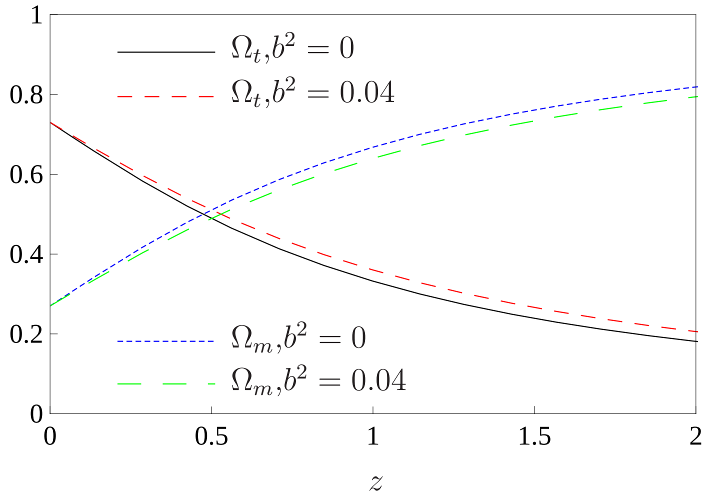

Figure 8.6: Variation of Ω_t and Ω_m with respect to the redshift for the holographic tachyon model with and without interaction. We take in this plot $c = 1$ and $\Omega_{t,0} = 0.73$.

The analytical form of the potential in terms of the interacting holographic tachyon field cannot be determined due to the complexity of the equations involved. However, we can obtain it numerically. The reconstructed $V(\phi)$ is plotted in Figs. 8.2 and 8.4. The scalar field $\phi(z)$ is also reconstructed by solving Eq. (8.24) and shown in Figs. 8.1 and 8.3. Selected curves are plotted for the cases of $c = 1$ and $b^2 = 0, 0.02, 0.04$ and 0.06 in Figs. 8.1 and 8.2. And for the cases of $b^2 = 0.02$ and $c = 0.9, 1.0, 1.1$ and 1.2 in Figs. 8.3 and 8.4. The present fractional matter density is chosen to be $\Omega_{m,0} = 0.27$. Figs. 8.1 to 8.4 display the dynamics of the interacting tachyon scalar field explicitly. The SN Ia observations have provided information of the cosmic expansion history around the redshift $z \leq 2$ by the measurement of luminosity distances of the sources. Therefore, we have taken the redshift range between $z = 0$ and $z = 2$. Following the interacting holographic evolution of the Universe, all the potentials are more steep in the early epoch, tending to be flat near today. Consequently, the tachyon field ϕ rolls down the potential more slowly as the universe expands (the kinetic term $\dot{\phi}^2$ gradually decreases) and the equation of state parameter tends to negative values close to -1 according to Eq. (8.8) as $\dot{\phi} \to 0$. As a result $dw_t/d\text{lna} < 0$. Note that $\phi(z)$ increases with z but becomes finite at high redshift. This means that ϕ decreases as the universe expands.

A somewhat similar behaviour was obtained in [169] for a holographic tachyon model. This was to be expected because the coupling that gauges the interaction in the IHDE model is small, otherwise this model would deviate significantly from the concordance model, making it incompatible with observations [156, 157].

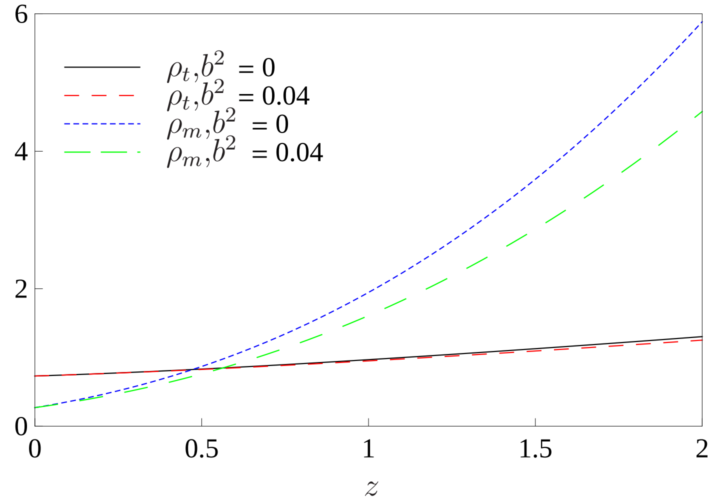

Figure 8.7: Variation of ρ_t and ρ_m with respect to z in units of $\varrho_{cr,0}$ for the holographic tachyon model with and without interaction. We take in this plot $c = 1$ and $\Omega_{t,0} = 0.73$.

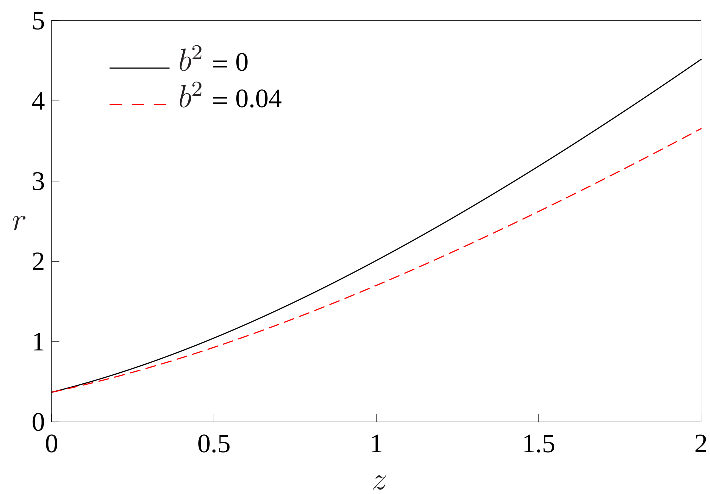

Figure 8.8: Variation of the ratio $r \equiv \rho_m/\rho_t$ with respect to the redshift for the holographic tachyon model with and without interaction. We take in this plot $c = 1$ and $\Omega_{t,0} = 0.73$.

Fig. 8.6 shows the impact of the interaction between HDE and dark matter, namely, Ω_t increases at a faster rate as compared to the non-interacting case. In addition, from Fig. 8.7 we learn that the point where ρ_t and ρ_m cross, $\rho_t = \rho_m$, occurs earlier in the interacting scenario. This latter feature is appreciated in more detail in Fig. 8.8 where the dependence of the ratio $r \equiv \rho_m/\rho_t$ with respect to the redshift z is depicted. The aforementioned ratio decreases monotonously with the expansion and varies slowly at the present epoch, decreasing slower when the interaction is considered. This implies that in this scenario the coincidence problem gets alleviated and besides, that dark energy is decaying into dark matter in recent epochs. Strictly speaking, it does not solve the coincidence problem in full because the model cannot predict that $r \sim O(1)$. Nevertheless, to the best of our knowledge, no model is able to predict that.

8.5 Conclusions

Interacting dark energy models were introduced to alleviate the coincidence problem. The extra degree of freedom relaxes the fine tuning of initial conditions present in the ΛCDM model. Consistency with the second law of thermodynamics requires the coupling be such that the overall transfer of energy goes from dark energy to dark matter.

In this chapter, we have proposed an interacting holographic tachyon model of dark energy with the future event horizon as infrared cut-off. This has been done by establishing a correspondence between the energy densities of the IHDE model and the tachyon field.

By assuming that the scalar field models of dark energy are effective theories of an underlying theory of dark energy, represented here by the IHDE, we can use the tachyon scalar field model to mimic the evolving behaviour of the IHDE. As a result, we have reconstructed the potential and the dynamics of the interacting holographic tachyon model in the region $-1 < w_t < 0$, i.e. before the phantom crossing, which is the allowed region for the tachyon field.

In summary, we have shown that the interacting holographic evolution of the universe can be completely described by a tachyon scalar field as it has proven viable when contrasted with observations, i.e. SN Ia, CMB, large scale structure, $H(z)$ and age constraints, clearly alleviating the coincidence problem and showing no tension with the age of the APM 08279 + 5255 quasar and the oldest globular cluster M 107.

Conclusions

The main results presented in this thesis are the following:

- Essentially, there are three distinct possible causes for the observed acceleration of the universe: (i) the existence of an anti-gravitational dark energy which is currently described in terms of a scalar field pervading the whole universe, (ii) an important modification of general relativity itself amounting to the addition of an extra term in the gravitational Lagrangian, and (iii) the very cosmic entanglement energy (physically equivalent to a Bohmian sub-quantum potential) which, in order to describe an accelerating universe, makes superfluous the inclusion of any dark energy fluid or field and requires no change whatsoever of the background gravitational theory, which continues being Einstein's general relativity proper. The latter option has been taken in this work to represent a so-called benigner phantom model that corresponds to an equation of state with tracking parameter w slightly smaller than -1 whereas it is described by a conventional kinetic term, possesses no violent instabilities at all and does not show any future big rip. The classical limit of such a model is simply the de Sitter universe.

- The main simplest cosmic solutions for a universe endowed with quantum entanglement entropy are fundamentally two: one corresponding to an equation of state with $w > -1$ and the other with the above mentioned benigner phantom behaviour. Thermodynamic arguments led us to conclude that only the second of such solutions satisfies the second law and therefore, it is physically feasible. That solution is free from the shortcomings plaguing current phantom cosmologies and has another remarkable virtue: it predicts the existence of a cosmic holographic surface placed at the Hubble horizon, a property which is demanded by most theoreticians but that very few models are able to fulfil.

- The most recent observational data for the cosmic equation of state parameter w predict a value that it is very close to -1, as corresponding to the case of a cosmological constant, showing perhaps an observable bias towards slightly smaller values. Since the quantum benigner phantom model precisely describes a situation where the universe corresponds to a de Sitter space with a cosmological constant plus a very small time-dependent perturbation, one can always adjust that model to the observations with an unbounded accuracy.

- The occurrence of a scaling accelerating phase after matter dominance has been shown to be rather problematic for most of the existing dark energy and modified gravity models although this is not actually the case for the benigner phantom model. In the quantum cosmic scenario corresponding to an entangled-energy driven accelerated model, the effective mass that can be associated with the matter particles has been shown to vanish just at the coincidence time, so that a cosmic system where the matter dominance phase is naturally followed by accelerating expansion is fully allowed.

- We have generalised the above thermodynamically allowed quantum cosmic solution in two respects: on the one hand, it can be described in a multidimensional realm by using a procedure similar to that followed in the de Sitter space. On the other hand, the model can be equivalently accommodated to include another quantum-mechanical aspect by using the methodology of Euclidean gravity and this finally endowed the resulting scenario with semi-classical entropy and temperature. Observers inside the event horizon of such a space would detect an isotropic flow of thermal radiation similar to the one emitted in a de Sitter space. There is another kind of radiation in this multidimensional space-time. It corresponds to gravitational waves which have been obtained by studying the Liftshift-Khalatnikov tensor perturbations. It is seen that these waves do not destabilise the space even though such a space might still develop a semi-classical instability.

- The cosmic benigner phantom scenario can be considered to describe a purely quantum single accelerating universe which has no classical analog nor future big rip singularity, much in the same sense as how the nucleation of baby universes branching off from an accelerating universe actually implies. That cosmic equivalence between essential quantum nature and accelerating expansion might perhaps be extended to also encompass the very concept of life whose origin has been many times stressed to be intimately related to a certain quantum phenomena and the existence of negative entropy, a situation which can most easily be accommodated to the concept of phantom energy.

- When one tries to adjust the benigner quantum model to Li's holographic description for dark energy, we can see that it by no means satisfies the fundamental relation between the energy density and the Hubble holographic parameters required by such a model when including a future event horizon. This leaves the Hubble horizon as the unique possible holographic screen.

- A connection between Li's holographic dark energy density and the kinetic k-essence energy density in flat FRW universe has been established. For the non-phantom case, this holographic dark energy can be described by a kinetic k-essence scalar field along a given procedure, a feature which has been shown in this work while reconstructing the kinetic k-essence characteristic function $F(X)$.

- A dilatonic description of the holographic dark energy has been derived by connecting the holographic dark energy density with the dilaton scalar field energy density in the simplest case of a flat FRW universe. We show that the predictions from the resulting model adjust perfectly well to the measured parameters for the observed accelerated expansion of our universe in the non-phantom case, so adequately reconstructing the kinetic term as well as the dynamics of the used dilaton scalar field.

- A model for a more general holographic dark energy tachyon model has been proposed in which we consider an interaction between the two usual components of the dark sector - dark matter and dark energy. The correspondence between the tachyon field and the holographic dark energy densities is used to reconstruct a phenomenological potential and the dynamics of the tachyon scalar field in a flat FRW universe. Such a model can also describe the observed accelerated expansion of the universe for a parameter space corresponding to the most recent observational results.

- The interacting holographic dark energy scenario displays a set of adjustable parameters large enough to become best adaptable to every observational situation even when one restricts oneself to the non-phantom sector. In particular, one would expect the interacting holographic tachyon model to satisfy a variety of choices for the relative contributions of dark matter and energy in the Padmanabhan's tachyon unification model.

Conclusiones

Los resultados principales que se han alcanzado con esta tesis son los siguientes:

- Esencialmente, hay tres posibles causas diferentes para la observada aceleración del universo: (i) la existencia de una energía oscura antigravitacional que se describe actualmente por medio de un campo escalar que domina el universo en su totalidad, (ii) una importante modificación de la relatividad general en sí misma equivalente a añadir un término extra en el lagrangiano gravitacional, y (iii) la propia energía cósmica de entrelazado (físicamente equivalente a un potencial subcuántico bohmiano), la cual, para describir la aceleración del universo, hace superflua la inclusión de cualquier fluido o campo de energía oscura y no requiere cambios en la teoría gravitacional de fondo, que continúa siendo la misma relatividad general de Einstein. En este trabajo se ha elegido esta última opción para representar el llamado modelo fantasma benigno, que corresponde a una ecuación de estado con parámetro de seguimiento w ligeramente más pequeño que -1, que, aunque es descrito por un término cinético convencional, no posee inestabilidades violentas y no muestra un gran desgarro futuro. El límite clásico de tal modelo es simplemente el universo de de Sitter.

- Las principales soluciones cósmicas más simples para un universo dotado con una entropía cuántica de entrelazado son fundamentalmente dos: una que corresponde a una ecuación de estado con $w > -1$ y la otra con el comportamiento fantasma benigno mencionado más arriba. Argumentos termodinámicos nos llevaron a concluir que sólo la segunda de tales soluciones satisface la segunda ley y, por lo tanto, es físicamente viable. Esa solución está libre de los defectos que asolan las cosmologías fantasma actuales y tiene otra notable virtud: predice la existencia de una superfie holográfica cósmica situada en el horizonte de Hubble, una propiedad que es demandada por la mayoría de los teóricos pero que muy pocos modelos son capaces de cumplir.

- Los datos observacionales más recientes para el parámetro de ecuación de estado cósmico w predicen un valor que está muy próximo a -1, lo que correspondería al caso de la constante cosmológica, mostrando quizá una tendencia observable hacia valores ligeramente más pequeños. Como el modelo cuántico cósmico benigno describe precisamente una situación donde el universo corresponde a un espacio de de Sitter con una constante cosmológica más una pequeñísima perturbación dependiente del tiempo, uno siempre puede ajustar ese modelo a las observaciones con una precisión ilimitada.

- Se ha mostrado que la existencia de una fase acelerante de escala después de una dominación de materia es bastante problemática para la mayoría de los modelos de energía oscura y gravedad modificada aunque este no es el caso del modelo fantasma benigno. Se ha mostrado que en el escenario cósmico cuántico que corresponde a un modelo acelerado impulsado por una energía de entrelazado, la masa efectiva que se puede asociar con las partículas de materia desaparece justo en el tiempo de coincidencia, con lo que un sistema cósmico donde la fase de dominación de materia es seguida de forma natural por una expansión acelerante está completamente permitido.

- Hemos generalizado la solución cósmica termodinámicamente permitida de más arriba en dos respectos: por un lado, se puede describir en un dominio multidimensional usando un procedimiento similar al seguido en el espacio de de Sitter. Por otro lado, el modelo se puede adaptar de forma equivalente para incluir otro aspecto mecanocuántico usando la metodología de la gravedad Euclidea y esto finalmente dotó al escenario resultante con una entropía y temperatura semiclásicas. Los observadores dentro del horizonte de sucesos de tal espacio detectarían un flujo isotrópico de radiación térmica similar al emitido en el espacio de de Sitter. Hay otro tipo de radiación en este espaciotiempo multidimensional. Corresponde a ondas gravitacionales que se han obtenido mediante el estudio de las perturbaciones tensoriales de Liftshift-Khalatnikov. Se ve que estas ondas no desestabilizan el espacio aunque dicho espacio podría todavía desarrollar una inestabilidad semiclásica.

- Se puede considerar que el escenario cósmico fantasma benigno describe un único universo acelerante cuántico que no tiene análogo clásico ni singularidad de gran desgarro futura, en el mismo sentido a como se hace en la nucleación de universos bebé que se desgajan de un universo acelerante. Esa equivalencia cósmica entre la naturaleza cuántica esencial y la expansión acelerante se puede tal vez extender para abarcar también el propio concepto de vida sobre cuyo origen se ha insistido muchas que veces que está íntimamente relacionado con ciertos fenómenos cuánticos y la existencia de entropía negativa, una situación que se puede adaptar fácilmente al concepto de energía fantasma.

- Cuando uno trata de ajustar el modelo cuántico benigno a la descripción holográfica de Li para la energía oscura, podemos ver que en ningún modo se satisface la relación fundamental entre la densidad de energía y los parámetros holográficos de Hubble requeridos por tal modelo cuando se incluye un horizonte de sucesos futuro. Esto deja al horizonte de Hubble como la única pantalla holográfica posible.

- Se ha establecido una conexión entre la densidad de energía oscura holográfica de Li y la de la k-esencia cinética en un universo de FRW plano. Para el caso no fantasma, esta energía oscura holográfica se puede describir por un campo escalar cinético de k-esencia por medio de un procedimiento dado, un rasgo que se ha mostrado en este trabajo a la vez que se ha reconstruido la función característica $F(X)$ de la k-esencia cinética.

- Se ha obtenido un descripción dilatónica de la energía oscura holográfica conectando la densidad de energía oscura holográfica con el campo escalar dilatónico en el caso más simple de un universo de FRW plano. Mostramos que las predicciones del modelo resultante se ajustan perfectamente bien a los parámetros medidos para la aceleración de la expansión del universo observada en el caso no fantasma, reconstruyendo así adecuadamente tanto el término cinético como la dinámica de campo escalar dilatónico usado.

- Se ha propuesto un modelo de energía oscura holográfica taquiónica más general en el que se considera una interacción entre los dos componentes usuales del sector oscuro - materia oscura y energía oscura. Se ha usado la correspondencia entre la densidad de energía del campo taquiónico y la de la energía oscura holográfica para reconstruir un potencial fenomenológico y la dinámica del campo escalar taquiónico en un universo de FRW plano. Dicho modelo puede describir también la aceleración de la expansión del universo observada para un espacio de parámetros que corresponde a los resultados observacionales más recientes.

- El escenario de la energía holográfica interactuante muestra un conjunto de parámetros ajustables lo suficientemente grandes como para poder adaptarse a cualquier situación observacional incluso cuando uno se restringe al sector no fantasma. En particular, uno esperaría que el modelo holográfico taquiónico interactuante satisfaciera una variedad de posibilidades para las contribuciones relativas de materia y energía oscuras en el modelo de unificación taquiónico de Padmanabhan.

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